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Deterministic Area Coverage by UAV Swarms via Multi-Eulerian Rotor-Router Walks

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Abstract

The deterministic area coverage problems for UAV swarms based on multi-Eulerian rotor-router walks are investigated. The rotor-router model provides a quasi-random analogue of random walks with strong structural properties. The loop reversibility theorem is used; that is, under mild connectivity assumptions on the underlying discretized environment, a swarm of UAVs executing coordinated multi-Eulerian rotor-router walks achieves complete area coverage, in the sense that each directed edge is traversed exactly once by a single UAV. The results establish provable coverage guarantees that are robust to asynchronous execution. Furthermore, the loop reversibility theorem is generalized to the case of repeated coverage, where the area should be covered a prescribed number M of times, without requiring global synchronization among UAVs at the end of each traversal cycle. A multi-user and multi-agent platform has been developed to support mission planning and coordinated supervision of self-organizing UAV swarms.

Keywords: UAV swarm, Rotor-router model, Eulerian walk, Area surveillance, Self-organizing system.

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1. Introduction

Unmanned Aerial Vehicle (UAV) swarms have emerged as an effective technological solution for large-scale area surveillance, environmental monitoring, and distributed sensing tasks. In contrast to single-UAV systems, swarm-based approaches offer greater robustness, scalability, and flexibility, enabling coordinated coverage of complex and dynamic environments without reliance on centralized control. These advantages have motivated extensive research on decentralized coordination, collective motion, and coverage strategies for UAV swarms.

A broad range of swarm intelligence and bio-inspired methods [1] have been proposed for UAV coordination, including particle swarm optimization, ant colony optimization, and related heuristic approaches.

A communication and control model in collaborative UAV swarms is presented in [2]. The authors identify key requirements for autonomous coordination and communication to enable efficient and reliable cooperative task execution.

In [3], the authors introduce a collision-free formation tracking framework for multiple quadrotors under switching directed communication topologies, where UAVs coordinate using only local neighborhood information and a Hooke's-law-inspired collision avoidance mechanism, supported by Lyapunov stability analysis.

The paper [4] introduces a trajectory optimization framework for multiple UAVs performing collaborative assembly tasks, combining centralized planning with distributed coordination to improve efficiency and feasibility of cooperative motion. The path planning process is solved by using a novel central force optimization genetic algorithm.

In [5], the authors present a comprehensive survey of UAV path planning techniques, classifying existing approaches into classical, soft-computing, and hybrid methods, and analyzing their applicability, advantages, and limitations in complex environments.

The paper [6] provides a comprehensive review of swarm intelligence algorithms for multi-UAV collaboration, addressing collision avoidance, task assignment, path planning, and formation reconfiguration. The survey analyzes representative algorithms, discusses their advantages and limitations, and highlights current research trends and open challenges in UAV swarm systems.

Area coverage and persistent surveillance by UAV swarms are fundamental problems in autonomous systems. While randomized strategies are widely used, deterministic algorithms for dynamic environments with provable guarantees remain comparatively underexplored.

This paper addresses this gap by leveraging rotor-router dynamics (multi-Eulerian walks) [7] to guarantee full coverage. In this setting, UAVs follow simple local rules (rotating outgoing edges and moving accordingly) yet collectively produce globally coordinated exploration patterns. Such an approach naturally supports decentralized execution, collision avoidance, and automatic workload distribution, making it well-suited for swarm-based surveillance applications.

A detailed survey of the theoretical results and fundamental properties of the rotor-router model and the related Abelian sandpile (chip-firing) model is presented in [8]. The loop-reversibility theorem, probabilistic properties, and related results are presented in [9, 10, 11].

The generalized loop-reversibility theorem for the simultaneous motion of multiple particles (chips), proved in [12], establishes deterministic guarantees for the complete traversal of the underlying graph. This theorem formed the theoretical basis for the creation of a multi-user and multi-agent platform for self-organizing UAV swarm mission planning and supervision [13, 12, 14, 15]. This platform employs graph-based representations of the operational area, enabling multiple operators to collaboratively construct mission graphs, generate Eulerian traversal structures, and supervise execution. By compiling surveillance tasks into deterministic traversal plans rather than continuous trajectories, such systems ensure consistency between planning, simulation, and real-time execution, even in the presence of asynchronous UAV behavior or dynamic swarm reconfiguration.

In this paper, we develop a deterministic framework for area coverage by UAV swarms based on multi-Eulerian rotor-router walks. Leveraging loop reversibility properties of the rotor-router model, we establish provable guarantees of complete coverage under mild con-

nectivity assumptions on the discretized environment. Furthermore, the loop reversibility theorem is generalized to the case of repeated coverage, where the area is required to be traversed a prescribed number M of times, without the need for global synchronization between UAVs at the end of each traversal cycle. The proposed approach supports asynchronous execution and provides a mathematically rigorous alternative to heuristic and probabilistic coverage strategies for swarm-based surveillance.

2. Model and Preliminaries

Consider a directed graph (digraph) $G = (V, E)$ with a set of vertices $V = V(G)$ and a set of directed edges $E = E(G)$. A spanning subgraph G' of G is a digraph with the set of vertices $V(G') = V(G)$ and a set of edges $E(G') \subseteq E(G)$. A path of length n from vertex $a \in V$ to $b \in V$ is a sequence of distinct vertices v_1, v_2, \dots, v_{n+1} such that v_i and v_{i+1} are connected by an edge $e_i \in E$, $i = 1, 2, \dots, n$, $v_1 = a$, $v_{n+1} = b$. The path becomes a cycle if $a = b$. Edges that connect a vertex to itself are called self-loops (cycles of length 1). A cycle of length 2 consists of two adjacent vertices v_1, v_2 , which are connected by a pair of oppositely directed edges from v_1 to v_2 and back. We call such cycles dimers by analogy with lattice dimers (dominoes) covering two neighboring vertices. A cycle formed by more than two edges is called a contour.

An Eulerian circuit on a finite digraph is a walk that starts and ends on the same vertex and visits each directed edge exactly once. If such a walk exists, the digraph is called Eulerian. A digraph is strongly connected if, for any two distinct vertices v and w , there are paths from v to w and from w to v . A strongly connected digraph $G = (V, E)$ is Eulerian if and only if, for each vertex $v \in V$, the in-degree and out-degree of v are equal.

The rotor-router model is defined as follows. Consider an arbitrary connected digraph $G = (V, E)$. Denote the number of outgoing edges (out-degree) from the vertex $v \in V$ by d_v . The total number of edges of G is $|E| = \sum_{v \in V} d_v$. Each vertex $v \in V$ is associated with a rotor, which is directed along one of the outgoing edges from v . The rotor directions at the vertex v are specified by an integer variable α_v , which takes values from $0 \leq \alpha_v \leq d_v - 1$ for $d_v \geq 1$.

The set $\rho = \{\alpha_v | v \in V, 0 \leq \alpha_v \leq d_v - 1\}$ defines the rotor configuration. The state of the system at any time is defined by a rotor configuration together with the position of a chip at a vertex of G . At each time step, the chip arriving at a vertex v first changes the rotor direction from α_v to $(\alpha_v + 1)$, and then moves one step along the new rotor direction from v to the corresponding neighboring vertex. The rotor directions α_v are taken modulo d_v , that is, $\alpha_v \pm d_v \equiv \alpha_v$.

The rotor configuration ρ can be considered as a spanning subgraph of G ($\rho \subset G$) with the set of vertices $V(\rho) = V(G)$ and the set of directed edges $E(\rho) \subset E(G)$ coinciding with rotors. The state of the system at any moment is given by the pair (ρ, v) of the rotor configuration ρ and the position of the chip $v \in V$. A vertex $v \in V$ is called a sink if its out-degree $d_v = 0$. In the absence of sinks, i.e., when each vertex has at least one outgoing edge, the motion of the chip does not stop.

Now let us formulate the loop reversibility theorem for a multi-particle walk [12].

Consider a bidirected contour $C = (v_1, v_2, \dots, v_n)$ in a digraph $G = (V, E)$, that is, a contour in which the vertices v_i and v_{i+1} are connected by two edges, one in each direction, $1 \leq i \leq n$, where the n -periodicity is assumed, i.e., $v_{i \pm n} \equiv v_i$. Let e_i^+ and e_i^- be directed edges connecting v_i to v_{i+1} and v_i to v_{i-1} , respectively. Here, the n -periodicity is also assumed,

i.e., $e_{i\pm n}^+ \equiv e_i^+$ and $e_{i\pm n}^- \equiv e_i^-$. The superscripts at e_i^- and e_i^+ are introduced to denote the negative and positive directions at v_i with respect to the contour C , respectively. Given the rotor-router model defined on G , we say that the bidirected contour C obeys the *domino ordering* if for each vertex v_i , $1 \leq i \leq n$, there exists a rotor direction $\alpha_{v_i}^*$ such that the rotor $\alpha_{v_i}^*$ is oriented along the edge e_i^- and $\alpha_{v_i}^* + 1$ is oriented along the edge e_i^+ . The directions $\alpha_{v_1}^*, \dots, \alpha_{v_n}^*$ are called negative with respect to C , whereas the directions $\alpha_{v_1}^* + 1, \dots, \alpha_{v_n}^* + 1$ are called positive, respectively.

Theorem 1. (See [12]). *Given an arbitrary finite Eulerian digraph G , let $C = (v_1, \dots, v_n)$ be a bidirected contour obeying domino ordering. Let $k < n$ rotor-router particles start their motion at contour vertices v_1, \dots, v_n , so that there is no more than one particle at one vertex. Denote these vertices as v_{S_1}, \dots, v_{S_k} . Assume that the rotors v_i , ($i = 1, 2, \dots, n$) at the contour C initially have positive directions. During the multi-particle walk, if a particle arrives at one of the vertices v_{S_j} , ($j = 1, 2, \dots, k$) with a negatively directed rotor at that vertex, that particle stops its motion. Then, after some number of steps, regardless of the order in which the particles move, the walk produces a configuration with negative directions $\alpha_{v_1}^*, \dots, \alpha_{v_n}^*$. Moreover, all vertices v_{S_1}, \dots, v_{S_k} will be occupied by one particle each. Assuming the initial rotor-router configuration is recurrent, namely, there are no other cycles except C , each rotor internal to C will perform a full rotation at the end of the walk.*

We now generalize this theorem to the case in which the area is required to be covered a prescribed number M of times, without the need for global synchronization between UAVs at the end of each traversal cycle. To establish this property, we construct M copies of the graph G that coincide at their vertex sets. Applying the above theorem to the resulting graph implies that all of its edges are traversed. Consequently, this construction is equivalent to a rotor-router walk on the original graph G , in which the contour C is reversed M times, and each edge is traversed exactly M times.

3. Application for UAV Swarm Coverage

The above result has a direct interpretation in the context of UAV swarm-based area coverage. In the multi-Eulerian rotor-router framework, repeated coverage with multiplicity M may lead to the accumulation of multiple UAVs at certain vertices of the discretized environment graph. For a vertex v_i with a degree d_i , the maximal number of UAVs that may simultaneously accumulate at v_i is bounded by $M \times d_i$, which grows linearly with the required coverage multiplicity M . While such accumulation is unavoidable in deterministic multi-agent traversal, it can be handled in a controlled manner.

To address this issue, we propose maintaining three-dimensional LIFO (last-in-first-out) stacks of UAVs at each vertex. These stacks provide a collision-free mechanism for temporarily storing multiple UAVs at the same spatial location while preserving the logical structure of the rotor-router dynamics. Furthermore, to avoid collisions during traversal along edges, we introduce an edge-priority rule for UAVs moving in intersecting or opposite directions, ensuring conflict-free motion without global coordination.

Since all UAVs are identical and the rotor-router walk possesses the Abelian property, neither the temporary accumulation of UAVs at vertices nor any permutation of UAV identities affects the final traversal state. Consequently, the proposed stacking and priority mechanisms preserve the deterministic coverage guarantees established by the theory, while

enabling practical realization of repeated area coverage by UAV swarms in three-dimensional space.

4. Multi-User and Multi-Agent Platform: Overview and Operating Regimes

To support the practical deployment of the proposed deterministic coverage algorithms, a multi-user and multi-agent platform for mission planning, execution, and supervision of self-organizing UAV swarms is developed. The platform provides a unified environment in which multiple operators can collaboratively define surveillance tasks, construct discrete mission representations, and monitor swarm execution in real time, while the UAVs themselves operate as autonomous agents executing deterministic traversal rules.

The platform is based on a graph-centric representation of the operational area, where the environment is discretized into a digraph $G = (V, E)$. Vertices represent mission-relevant locations or states, and edges encode admissible traversal paths. This representation is shared consistently across all connected users and UAV agents, ensuring that planning, simulation, and execution rely on the same underlying model. Deterministic compilation of multi-Eulerian traversal tasks allows mission specifications to be translated into reproducible swarm behaviors without dependence on probabilistic decision-making.

From the operator's perspective, the platform enables collaborative mission authoring, where multiple users can simultaneously participate in defining, modifying, and supervising the mission graph. From the UAV perspective, each agent executes local rotor-router rules using only local information, while the collective swarm behavior realizes global coverage objectives derived during mission planning.

The platform supports several operating regimes that correspond to different stages of the mission lifecycle:

1. **Mission Preparation Regime.** In this regime, users collaboratively construct and edit the mission graph, assign roles or constraints to vertices and edges, and specify coverage requirements, including repeated coverage with a prescribed multiplicity M . Additionally, all directed edges are ordered using a fixed priority to organize the above-discussed collision-free mechanism. Deterministic compilation procedures are applied to transform the mission graph into traversal structures compatible with rotor-router execution.

2. **Simulation and Validation Regime.** Before deployment, the compiled traversal plans are executed in a simulation environment. This regime allows users to validate coverage completeness, detect potential conflicts, and assess UAV accumulation behavior at vertices (stack sizes).

3. **Execution and Supervision Regime.** During mission execution, UAVs operate as autonomous agents following rotor-router dynamics. The platform provides supervision and visualization of swarm progress, enabling users to observe traversal states, vertex occupancies, and mission completion without intervention in local UAV decision-making.

4. **Adaptive Reconfiguration Regime.** The platform accommodates dynamic changes such as UAV entry or exit, provided that the underlying graph connectivity conditions are preserved. Due to the Abelian property of the rotor-router model, such reconfigurations do not affect the final coverage outcome, allowing mission continuity without global synchronization.

A key advantage of the platform is the separation between multi-user coordination at the planning and supervision level and multi-agent autonomy at the execution level. Users interact with high-level mission abstractions, while UAVs execute simple, local, deterministic

rules. This separation ensures that concurrent user interactions, UAV permutations, or asynchronous execution do not alter the final coverage guarantees, making the platform well-suited for scalable and reproducible UAV swarm surveillance missions.

A demonstration video illustrating the proposed platform is provided in [16].

5. Conclusion

This paper presented a deterministic framework for area coverage by UAV swarms based on multi-Eulerian rotor-router walks. Using loop reversibility properties, we established provable guarantees for complete and repeated coverage that are robust to asynchronous execution and independent of probabilistic assumptions. The theoretical results were further interpreted in the context of UAV swarm deployment, where UAV accumulation at vertices was shown to scale linearly with the coverage multiplicity and handled using collision-free stacking and priority mechanisms.

In addition, the proposed model naturally supports integration with a multi-user and multi-agent platform for UAV swarm mission planning and supervision, enabling deterministic task compilation, coordinated execution, and consistent supervision across multiple operators. The presented approach provides a rigorous alternative to heuristic swarm coordination methods and forms a foundation for scalable and reproducible UAV-based surveillance systems.

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Տարածքի դետերմինացված ծածկումը ԱԹՍ երամի կողմից՝ բազմամասնիկային էյլերյան rotor-router դեզերմամբ

Վահագն Ս. Պողոսյան

ՀՀ ԳԱԱ Բնֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտ,
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Անփոփում

Ուսումնասիրվել են տարածքի դետերմինացված ծածկման խնդիրները ԱԹՍ երամների համար՝ հիմնված բազմամասնիկային էյլերյան rotor-router դեզերումների վրա:

Rotor-router մոդելը հանդիսանում է պատահական դեգերումների քվազի-պատահական անալոգը՝ օժտված հստակ կառուցվածքային հատկություններով:

Կիրառվել է կոնտուրի շրջելիության թեորեմը, ըստ որի՝ դիսկրետացված միջավայրի կապակցվածության որոշակի պայմաններում, համակարգված բազմամասնիկային էյլերյան rotor-router դեգերումներ իրականացնող ԱԹՍ երամը ապահովում է տարածքի ամբողջական ծածկում՝ այն իմաստով, որ յուրաքանչյուր ուղղորդված կողով իրականացվում է ճիշտ մեկ անգում միայն մեկ ԱԹՍ-ի կողմից:

Ստացված արդյունքները սահմանում են ապացուցելի ծածկման երաշխիքներ, որոնք կայուն են ասինխրոն կատարման նկատմամբ:

Կոնտուրի շրջելիության թեորեմը ընդհանրացվել է բազմակի ծածկման դեպքի համար, երբ պահանջվում է տարածքը ծածկել նախապես տրված M անգամ՝ առանց յուրաքանչյուր դեգերման ցիկլի ավարտին ԱԹՍ-ների միջև գլոբալ սինքրոնացման անհրաժեշտության:

Մշակվել է բազմաօգտատեր և բազմաազենոս հարթակ, որն ապահովում է ինքնակազմակերպվող ԱԹՍ երամների առաքելությունների պլանավորումը և համակարգված վերահսկումը:

Բանալի բառեր՝ ԱԹՍ խումբ, rotor-router մոդել, էյլերյան քայլ, տարածքի հսկում, ինքնակազմակերպվող համակարգ:

Детерминированное покрытие области роём БПЛА с использованием многочастичных эйлеровых rotor-router блужданий

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Аннотация

Исследованы задачи детерминированного покрытия области роями беспилотных летательных аппаратов (БПЛА), основанные на многочастичных эйлеровых rotor-router блужданиях.

Модель rotor-router представляет собой квазислучайный аналог случайных блужданий, обладающий выраженными структурными свойствами.

Использована теорема обратимости контуров, согласно которой при определенных условиях связности дискретизированной среды рой БПЛА, выполняющий согласованные многочастичные эйлеровы rotor-router блуждания, обеспечивает полное покрытие области в том смысле, что для каждого ориентированного ребра осуществляется ровно один проход только одним БПЛА.

Полученные результаты устанавливают доказуемые гарантии покрытия, устойчивые к асинхронному выполнению. Кроме того, теорема обратимости контуров обобщается на случай многократного покрытия, когда требуется покрыть область заданное число M раз без необходимости глобальной синхронизации между БПЛА в конце каждого цикла блуждания.

Разработана многопользовательская и многоагентная платформа, обеспечивающая планирование миссий и координированный надзор за самоорганизующимися роями БПЛА.

Ключевые слова: рой БПЛА, модель rotor-router, эйлерово блуждание, наблюдение территории, самоорганизующаяся система.