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PDE-UNet: A Modified UNet Architecture Applied to Medical Image Segmentation

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Abstract

Medical image segmentation is a critical task in healthcare, particularly for disease detection and proper treatment planning. Deep learning models achieve high performance in medical image analysis. This paper presents the effectiveness of the new PDE-UNet architecture, inspired by the applications of partial differential equations (PDEs) in neural networks, to enhance medical image segmentation performance. Experiments were conducted on brain tumor MRI images from the BraTS2020 dataset and compared with the traditional UNet architecture.
Keywords: Medical Image Segmentation, Brain Tumor Segmentation, UNet, PDE-inspired CNN block, MICCAI BraTS 2020 Challenge.
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1. Introduction

Artificial intelligence and deep learning models are becoming highly efficient in healthcare [1], providing automated analysis, diagnosis, and decision support. Particularly, they demonstrated performance in medical imaging tasks, such as segmentation, classification, and detection [2].

PDEs play an important role in image processing and computer vision [3]. They have been widely used for edge detection [4], image denoising [5], and image inpainting [6]. By modeling continuous transformations, PDEs enable the preservation of important structures while effectively reducing noise and improving image quality.

UNet is a convolutional neural network (CNN) architecture developed for medical image analysis, which can be trained on relatively small datasets and achieve high performance [7]. Today, it remains a primary tool for segmentation tasks in medical imaging. The UNet consists of two parts: an encoder and a decoder, connected by skip connections, as shown in Fig.

The primary goal of this study is to investigate the power of PDE-UNet in medical image segmentation tasks, with a particular focus on brain tumor segmentation using the

BraTS2020 dataset [8] from the MICCAI BraTS 2020 challenge. PDE-UNet proposes a trainable preprocessing PDE-inspired convolutional block to extract important image features before passing the input to the main segmentation network.

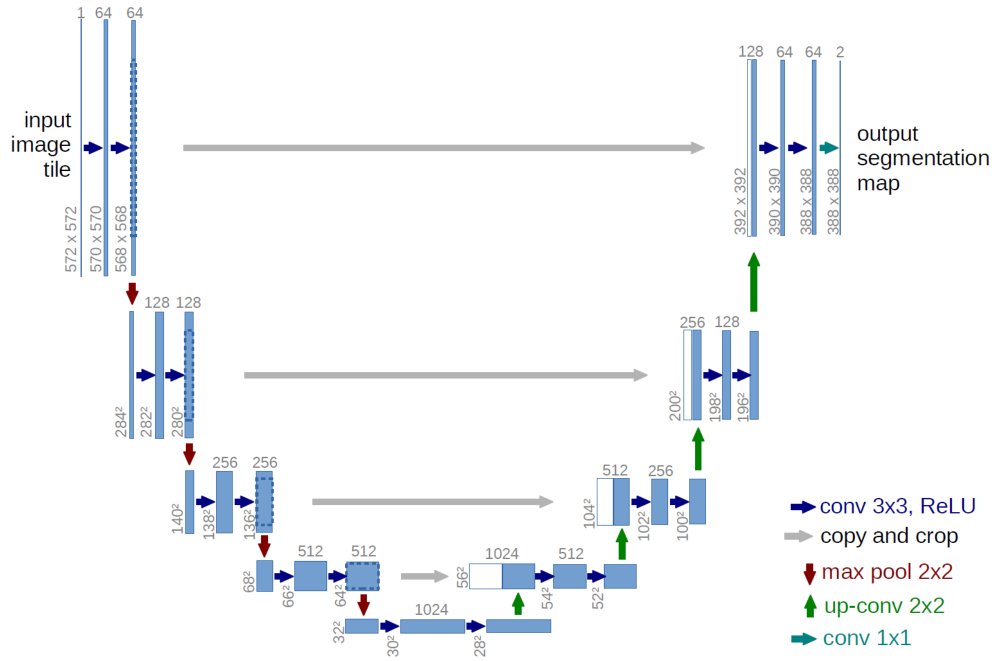

Fig. 1. UNet architecture.

2. Related Work

2.1 Medical Image Segmentation using Deep Learning

Deep learning has achieved significant success in medical image analysis, enabling the automation of clinical tasks such as tumor detection and lesion segmentation. Despite various approaches, encoder-decoder architectures such as UNet have become dominant in the field, demonstrating high performance even with small datasets.

2.2 Partial Differential Equations in Neural Networks

Due to the wide use of PDEs in image processing, they have been integrated into deep neural networks [9] to improve feature representation, stability, and local interaction modeling [10], as well as to preserve structural information. An example of such integration is demonstrated in [11], where the integration of the Cahn-Hilliard PDE-based model into the UNet architecture for image segmentation was proposed.

Recent research [12] has integrated a PDE-inspired convolutional operator, based on the Cable equation (1), which models the transmission of potentials in neural cells of the human brain [13], as a trainable preprocessing layer in a residual CNN for classification tasks.

$$\tau_m \frac{\partial v(x,t)}{\partial t} = -v(x,t) + \lambda_m^2 \frac{\partial^2 v(x,t)}{\partial x^2} + r_m I_{ext}(x,t), \;\; t \geq 0, \tag{1}$$

The final form of the discretized PDE operator, using the finite difference method [14] for equation (1), is as follows.

$$u_{i,k}^{t+1} = (1-\tau)\cdot u_{i,k}^{t} + (P_1 + P_2)\cdot U, \tag{2}$$

where

$$U = \begin{bmatrix} u_{i-1,k-1}^{t} & u_{i-1,k}^{t} & u_{i-1,k+1}^{t} \\ u_{i,k-1}^{t} & u_{i,k}^{t} & u_{i,k+1}^{t} \\ u_{i+1,k-1}^{t} & u_{i+1,k}^{t} & u_{i+1,k+1}^{t} \end{bmatrix}, P_1 = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\Phi^2} & -\frac{2}{\Phi^2} & \frac{1}{\Phi^2} \\ ‘0 & 0 & 0 \end{bmatrix}, P_2 = \begin{bmatrix} 0 & \frac{1}{\Psi^2} & 0 \\ 0 & -\frac{2}{\Psi^2} & 0 \\ 0 & \frac{1}{\Psi^2} & 0 \end{bmatrix}.$$

P_1 and P_2 are defined as two-dimensional weighted convolution operators for the neural network with weights Φ, Ψ, and τ.

2.3 Motivation

While advanced architectures such as Attention UNet [15] and TransUNet [16] perform well in medical image segmentation, they often struggle to capture fine structural details and typically need large, annotated datasets. Motivated by previous research in image classification, we investigate the effectiveness of PDE-UNet in medical image segmentation, aiming to enhance feature extraction and segmentation accuracy, and recommend adding a PDE-inspired lightweight preprocessing layer to the traditional UNet.

3. Experiments

All models were implemented in PyTorch and trained on a NVIDIA RTX 3050 GPU with 4 GB of memory. Our experiments focus on brain tumor MRI images from the BraTS2020 dataset, part of the MICCAI BraTS2020 challenge. A sample of the dataset is shown in Fig. 3. The dataset consists of multi-modal brain MRI scans, including FLAIR, T1, T1ce, and T2 modalities, with expert annotations for three tumor subregions: the enhancing tumor (ET), tumor core (TC), and whole tumor (WT). The experiments were made on the extracted 2D slices from the volumes, using the middle slices of each modality, which still capture the most relevant tumor structures.

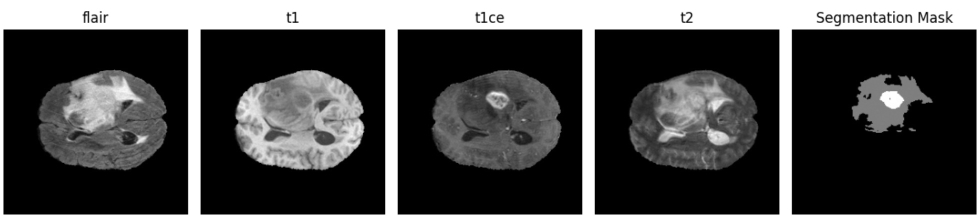

Fig. 2. A sample from the BraTS dataset.

We conducted experiments on a UNet architecture, which has skip connections linking the encoder and decoder layers. This design aligns well with our PDE-inspired convolutional block, which also incorporates a skip connection, allowing for effective feature propagation and preservation. In this study, the UNet architecture takes 4-channel input images corresponding to the FLAIR, T1, T1ce, and T2 MRI modalities. The encoder consists of convolutional blocks with increasing feature dimensions of 32, 64, 128, and 256 channels, followed by a bottleneck block with 512 features. The decoder mirrors the encoder, using transposed convolutions to upsample the feature maps, and combines these with skip connections from the corresponding encoder layers. The final 11 convolution produces four output channels corresponding to the tumor subregions (background, ET, TC, and WT). The proposed PDE-UNet architecture has an additional PDE-inspired block compared with UNet. Figure 3. illustrates the UNet and PDE-UNet architectures.

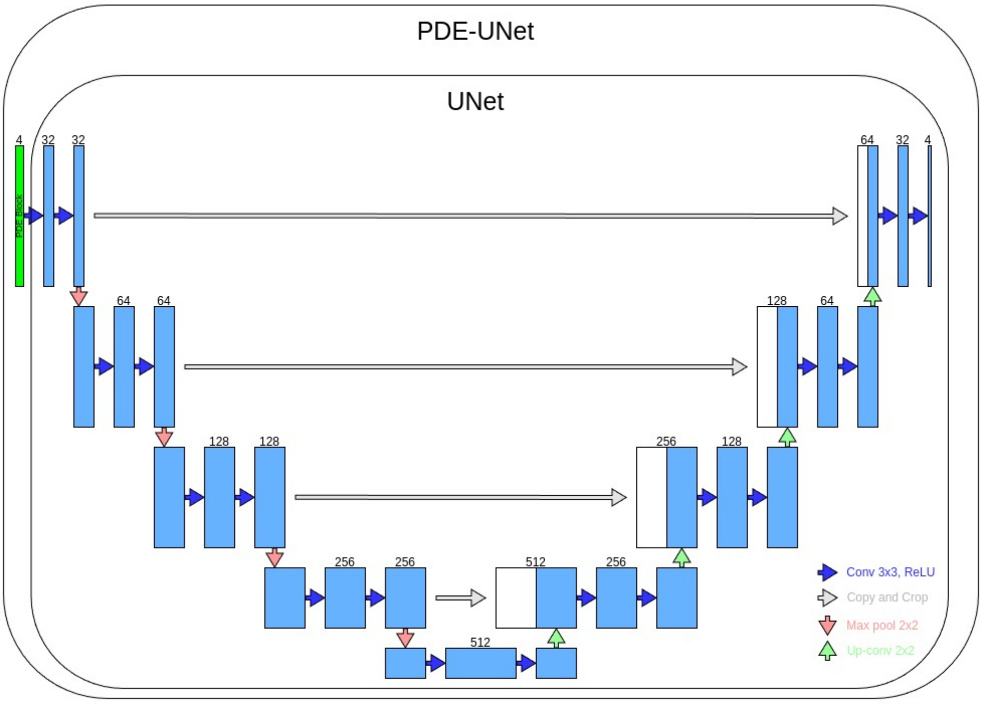

Fig. 3. The UNet and PDE-UNet architectures.

For the optimization process, the Adam optimizer [17] was used with a one-cycle learning rate scheduler [18]. As a loss function, a combo loss [19] combining the cross-entropy loss [20] and the dice loss [21] was used to optimize the medical image segmentation model. The cross-entropy provides stable and smooth gradient propagation, facilitating consistent optimization, while the dice loss effectively handles class imbalance problems and improves the segmentation of small regions.

The Cross-Entropy loss is defined as:

$$\mathcal{L}_{CE} = -\frac{1}{N}\sum_{i=1}^{N}\sum_{c=1}^{C} y_{i,c}\log(\hat{y}_{i,c}), \tag{3}$$

where N is the number of pixels, C is the number of classes, $y_{i,c}$ is the ground-truth label for class c at pixel i, and $\hat{y}_{i,c}$ is the predicted probability for class c at pixel i.

The Dice loss is defined as:

$$\mathcal{L}_{Dice} = 1 - \frac{2\sum_{i=1}^{N}\sum_{c=1}^{C} y_{i,c}\hat{y}_{i,c} + \epsilon}{\sum_{i=1}^{N}\sum_{c=1}^{C} y_{i,c}^2 + \sum_{i=1}^{N}\sum_{c=1}^{C} \hat{y}_{i,c}^2 + \epsilon}, \tag{4}$$

where ϵ is a small constant to avoid division by zero.

The final loss function is the sum of both

$$\mathcal{L} = \mathcal{L}_{CE} + \mathcal{L}_{Dice}, \tag{5}$$

Training was performed for 10 epochs with a batch size of 8. The dataset was divided into training, validation, and test sets in an 80%-10%-10% split. We applied several data augmentation techniques to increase the diversity and robustness of our training dataset. These techniques included random horizontal and vertical flips, rotations, affine transformations (shifts and scaling), and adjustments to brightness and contrast. Such augmentations are commonly used in medical image segmentation tasks to reduce overfitting and improve model generalization [22]. Model performance was evaluated using the Dice coefficient and Intersection over Union (IoU) metrics.

4. Results

The training progress of both models, UNet and PDE-UNet, was analyzed by monitoring the evolution of the training and validation losses. Both the baseline and the proposed models showed steady convergence during the training process. The PDE-UNet reached a lower validation loss compared to the standard UNet. Fig. 4. illustrates the progression of loss for both models over epochs.

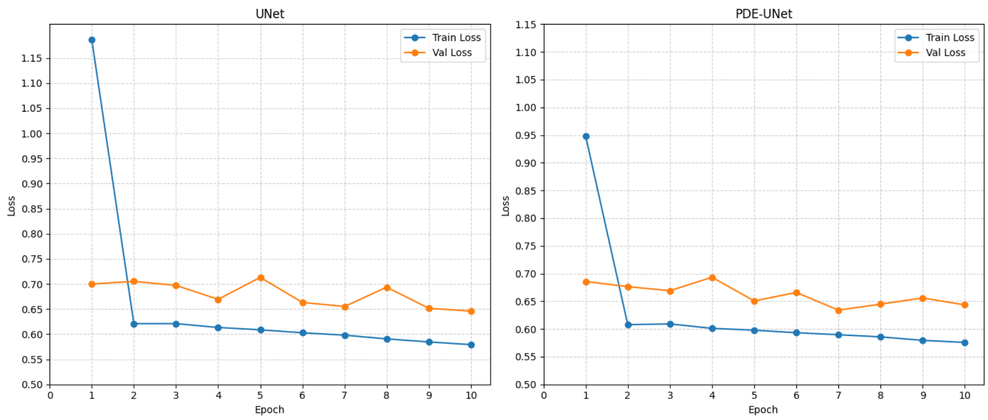

Fig. 4. Left: The training process of the UNet. Right: The training process of the proposed PDE-UNet.

Table 1. Results of Dice coefficient and IoU metrics evaluated on the validation and test sets for the UNet and PDE-UNet models

Model	**Validation**		**Test**		**Parameter count**
	Dice	**IoU**	**Dice**	**IoU**	
UNet	0.438	0.389	0.455	0.372	7,766,372
PDE-UNet	0.462	0.406	0.469	0.394	7,766,628

We evaluated segmentation performance using the Dice coefficient and IoU metrics. The evaluation was performed using the checkpoint from epoch 10 for UNet and epoch 7 for

PDE-UNet. The results are presented in Table 1. On average, the PDE-UNet achieved higher Dice and IoU scores than the baseline UNet, confirming its potential advantage in capturing structural details.

Fig. 5. shows segmentation outputs of both models alongside the ground truth annotation for the represented case. As we can see, the PDE-UNet produces more accurate and contiguous segmentations.

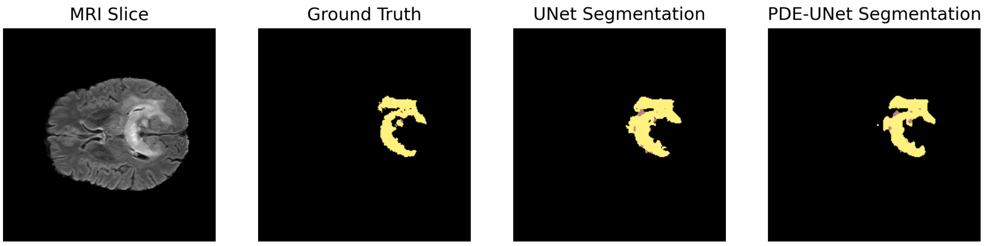

Fig. 5. The segmentation comparison between models.

It is interesting to observe how our trained PDE preprocessing block processes the images. Fig. 6. shows its effect on the sample images.

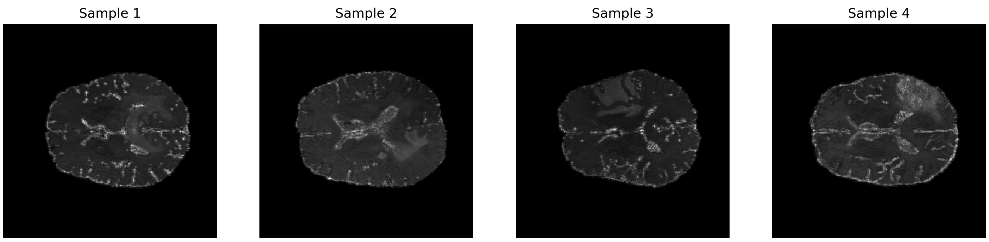

Fig. 6. Results of applying the trained PDE block to sample images.

Overall, these results demonstrate that the PDE-UNet with a PDE-inspired convolutional block enhances the representational power of the traditional UNet, leading to improved segmentation accuracy on the BraTS2020 dataset while adding only a small number of trainable parameters; therefore, the inference time of the model remains almost the same.

5. Conclusion

The paper recommends a new PDE-UNet architecture for medical image segmentation tasks. Motivated by previous research presenting a PDE-inspired preprocessing block, the recommended architecture includes an additional PDE block compared to the traditional UNet. The experiments were conducted on brain tumor segmentation using the BraTS2020 dataset. A comparative evaluation between PDE-UNet and the UNet demonstrates that the lightweight preprocessing block added to a PDE-UNet, with a few additional parameters, can positively impact the final segmentation quality, improving the UNet models. For future work, it would be interesting to extend the proposed block's investigation to other medical image analysis applications.

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PDE-UNet: Փոփոխված UNet ճարտարապետություն՝ կիրառված բժշկական պատկերների սեգմենտավորման համար

Ռաֆայել Մ. Վեզիրյան

ՀՀ ԳԱԱ Ինֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտ, Երևան, Հայաստան
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Ամփոփում

Բժշկական պատկերների սեգմենտացիան կարևորագույն խնդիր է առողջապահության ոլորտում, մասնավորապես հիվանդությունների հայտնաբերման և բուժման պատշաճ պլանավորման համար: Խորը ուսուցման մոդելները բարձր արդյունքների են հասնում բժշկական պատկերի վերլուծության մեջ: Այս հոդվածը ներկայացնում է նոր PDE-UNet ճարտարապետության արդյունավետությունը, որը ոգեշնչված է նեյրոնային ցանցերում մասնակի դիֆերենցիալ հավասարումների (ՄԴՀ) կիրառություններով, բժշկական պատկերի սեգմենտացիայի արդյունավետությունը բարելավելու համար: Փորձարկումները կատարվել են BraTS2020 տվյալների հավաքածուից վերցված ուղեղի ուռուցքի ՄՌՏ պատկերների վրա և համեմատվել են ավանդական UNet ճարտարապետության հետ:

Բանալի բառեր՝ բժշկական պատկերների սեգմենտացիա, ուղեղի ուռուցքի սեգմենտացիա, UNet, PDE-ով ոգեշնչված CNN բլոկ, MICCAI BraTS 2020 մարտահրավեր:

PDE-UNet: Модифицированная архитектура UNet, применяема для сегментации медицинских изображений

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Аннотация

Сегментация медицинских изображений критически важная задача в здравоохранении, особенно для диагностики заболеваний и планирования надлежащего лечения. Модели глубокого обучения достигают высокой эффективности при анализе медицинских изображений. В данной статье представлена эффективность новой архитектуры PDE-UNet, основанной на применении уравнений в частных производных (УЧП) в нейронных сетях, для повышения эффективности сегментации медицинских изображений. Эксперименты проводились на МРТ-изображениях опухолей головного мозга из набора данных BraTS2020 и сравнивались с традиционной архитектурой UNet.

Ключевые слова: сегментация медицинских изображений, сегментация опухолей головного мозга, UNet, блок CNN на основе PDE, конкурс MICCAI BraTS 2020.