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Fractal Geometry and Quantitative Evaluation of the Aesthetic Appeal of Ancient Armenian Architecture Monuments

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Abstract

A fractal analysis of the most important architectural monuments (temples) of medieval Armenia is given. Quantitative (objective) evaluations of their artistic appeal are obtained. These evaluations confirm the generally accepted (subjective) appeal of these temples as masterpieces of Armenian architecture. Based on the fractal data obtained, statistical conclusions are made about the high degree of architectural compatibility of the plans and facades of the monuments under consideration.
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1. Introduction

The basic figures of classical (Euclidean) geometry are simple and clear: circle, sphere, cylinder, pyramid, etc. The impressive achievements of this science allowed ancient thinkers to assume that the geometric picture of the world is described by Euclidean geometry based on the five Platonic solids (regular polyhedra).

Over time, it became clear that this position was only partly true, since it was impossible to describe the shapes of objects such as clouds, mountain ranges, coastlines of seas and lakes, etc. within the framework of classical geometry.

The question of the existence of a geometry that can describe and study the forms of such objects has been a topic of interest for scientists for a long time. However, it was only with the development of powerful computing systems (enabling us to visualize such structures) that the construction of such a theory became possible.

Geometry describing non-standard forms was proposed by B. Mandelbrot [1] based on the concept of fractal introduced by him. It was called fractal geometry.

The difference between this geometry and classical Euclidean geometry is as follows. In classical geometry, objects are idealized, meaning their surfaces are assumed to be perfectly smooth, without any irregularities, cracks, or breaks. In contrast, fractal geometry studies the patterns inherent in natural objects, processes, and phenomena with the presence of roughness, brokenness, and other complexities (see, for example, [2, 3]). It offers a variety of ways to describe and measure both natural and man-made objects.

Thanks to the development of fractal geometry, it has recently become possible to objectively (quantitatively) evaluate the aesthetic appeal of architectural compositions for the first time.

There are many monuments of world architecture for which fractal analysis has been carried out. These are remarkable Gothic cathedrals in Europe, beautiful mosques of Islamic architecture, unique Hindu temples (see, for example, [4, 5, 6, 7]). Such an analysis has not been carried out for Armenian temples. This paper attempts to fill this gap. It is devoted to the application of fractal geometry to the quantitative evaluation of the attractiveness of such outstanding architectural monuments of medieval Armenia as the Zvartnots, Hripsime and the Cathedral of the Holy Virgin in Ani.

When conducting fractal analysis, various computing tools are used. In this work, the analysis is carried out on the basis of the FrakOut! package, which is very convenient for calculating the fractal parameters of buildings. When evaluating the architectural compatibility of the plan and facade of the temples under consideration, the STATISTICA software package was used to find statistical estimates based on the available data.

2. Fractals and Fractal Geometry

According to Mandelbrot [1], a fractal is a structure consisting of parts that are in some sense similar to the whole (or to each other).

Fractals can be found almost everywhere in nature. For example, tree crowns, snowflakes, broccoli heads, crystals, etc.

From a mathematical point of view, a fractal is a geometric figure (a set of points in Euclidean space) whose fractal dimension (the Hausdorff–Besicovitch dimension) is either fractional or exceeds its topological dimension.

The Hausdorff-Besicovitch dimension of some finite set G, $G \subset \mathcal{R}^n$, is defined as follows. Consider an n-dimensional cubic lattice in $\mathcal{R}^n$ with the length of the edge of a cube (cell) equal to Δ. Let $N(\Delta)$ be the minimum number of cubes needed to cover the set G. Then the fractal dimension D of this set is defined based on the following requirement:

$$\lim_{\Delta \to 0} N(\Delta)\Delta^d = \begin{cases} 0, & \text{if } \ d > D, \\ \infty, & \text{if } \ d < D. \end{cases}$$

From this it is clear that the dimension D of the set G is essentially the boundary that shows that if $d < D$, then the number of cubes $N(\Delta)$ is insufficient to cover the set G, and if $d > D$, then the number of cubes $N(\Delta)$ is excessive for coverage.

It is generally accepted that the fractal dimension is a characteristic property of fractals, i.e., if the dimension D is not an integer, then the set G is considered a fractal. In practice, approximate numbers are used. For $\Delta \approx 0$, $D \approx -\ln N(\Delta)/\ln \Delta$.

In addition to natural fractals, there are also artificial (non-natural) fractals. The first examples of non-natural fractals were constructed at the end of the nineteenth century in connection with purely mathematical problems of function theory. From the point of view of classical mathematical analysis, they had extremely unusual properties. For example, this is the Cantor set (Cantor dust), the nowhere differentiable Weierstrass function, the Koch snowflake, the Brownian curve on the plane, etc. For some of them, fractal dimensions have been calculated: the Cantor set has a fractal dimension of $D = \ln 2/\ln 3$, and for a Brownian curve on a plane, it is equal to 2, that is, exceeds its topological dimension.

It should also be noted that fractal principles are present in the theory of fractional integro-differentiation as well. The fact is that in a fractal environment, the change in physical quantities can slow down to such an extent that it is impossible to describe such a process using an ordinary derivative. This can only be done using integro-differential equations that include a fractional derivative with respect to time. Armenian mathematicians M. Djrbashian and A. Nersessian made a significant contribution to this theory (see [8]).

3. Quantitative Evaluation of Aesthetic Appeal of Fractal Structures

Fractal geometry can be used as a method for analyzing the structure of buildings. It has been noted that if the fractal component of an architectural structure is clearly traced, then this structure has strong architectural aesthetics. Psychologists have developed a quantitative method to assess such aesthetics.

The first systematic studies of the perception of fractal forms were conducted by J. Sprott and his colleagues. These studies analyzed the relationships between objective (fractal dimension) and subjective assessments of the visual attractiveness of various objects (the results of these works are summarized in [9]). It was later shown that subjective assessments of visual attractiveness correlate quite strongly with fractal dimension and are reproduced upon repeated testing [10, 11, 12]. It was also shown that the fractal dimension is the main factor influencing subjective assessments of the attractiveness of objects with fractal properties. Preference is given to objects with an average fractal dimension in the range of 1.3–1.5 (flat images). Subsequently, many studies were devoted to the empirical study of the perception of fractals of natural and artificial origin (see, for example, [13, 10, 14, 15]).

Research by K. Hagerhall and her group [14] has established that emotional states in relation to natural landscapes can be predicted by typical fractal characteristics, i.e., by fractal dimension.

These studies confirmed the relationship between assessments of aesthetic appeal and complexity with fractal dimension.

4. Fractal Analysis of Armenian Temples

In architecture, fractal principles are used in the design of objects using computer modeling. These principles can be used to create unique and very interesting architectural forms (see, for example, [16]). In this case, practical methods for calculating the fractal dimension of the structures under consideration play an important role.

One of the most popular methods is the method of counting cells that have a non-empty intersection with the image being studied (box-counting dimension method). Apparently, W. Lorenz [17] and C. Bovill [18] were the first to study and use this method most fully.

Let us describe in general terms the algorithm for applying this method.

In the first step, a cubic (square) grid with the cell edge length (scale) equal to Δ is superimposed on the image under study. Initially, Δ is taken to be equal to L, where L is the length of the rectangle containing all the images. Let $N(\Delta)$ be the number of all cubes that have a non-empty intersection with the image under study.

Next, the following ration is considered

$$-\log N(\Delta)/\log \Delta,$$

and its behavior is investigated under stepwise changes in the scale Δ.

The scale is reduced by half at each step. The process can continue indefinitely, but in practical applications, it is stopped depending on the requirements of the task. The slope of the graph of $\log N(\Delta)$ from $-\log \Delta$ gives an approximate value of the fractal dimensions of the image.

Below, we will present a fractal analysis of the temples of Hripsime and Zvartnots, as well as the Ani Cathedral, using the FrakOut! program. In parallel, a statistical analysis of the compatibility of the plan and facade of these buildings is also carried out.

The results obtained show that the temples under consideration have high architectural attractiveness, and their plan and facade are in excellent agreement with each other.

4.1 Hripsime

The temple was built by Catholicos Komitas in 618 to the east of Echmiadzin on the burial site of Saint Hripsime. It is a central-domed structure with an internal cross-shaped base. It is a recognized masterpiece of Armenian architecture.

Appendix 1 contains fragments of the process of calculating the fractal dimension of the facade and plan of the Hripsime temple. The results of the calculations are summarized in Table 1.

Table 1. Fractal dimension of the Hripsime Cathedral facade and plan

Calculation of fractal dimension between:		fractal dimension	
large grid size	small grid size	facade	plan
200	100	1.46	1.74
100	50	1.48	1.58
50	25	1.49	1.49
25	12.5	1.49	1.51
general fractal dimension		1.48	1.58

From the obtained data, it follows that the temple of Hripsime has an average fractal dimension of 1.48. The calculations also show that the standard deviation of these data from the average is 0.014. Regarding the architectural plan, the following estimates were obtained: the average fractal dimension is 1.58 with a standard deviation of 0.113. The correlation between the fractal dimensions of the facade and the plan is -0.997.

Fig. 1 shows a graph of the dependence of $\log N(\Delta)$ on $-\log \Delta$ for the facade of the temple, which is a linear regression constructed using the obtained values of the fractal dimension.

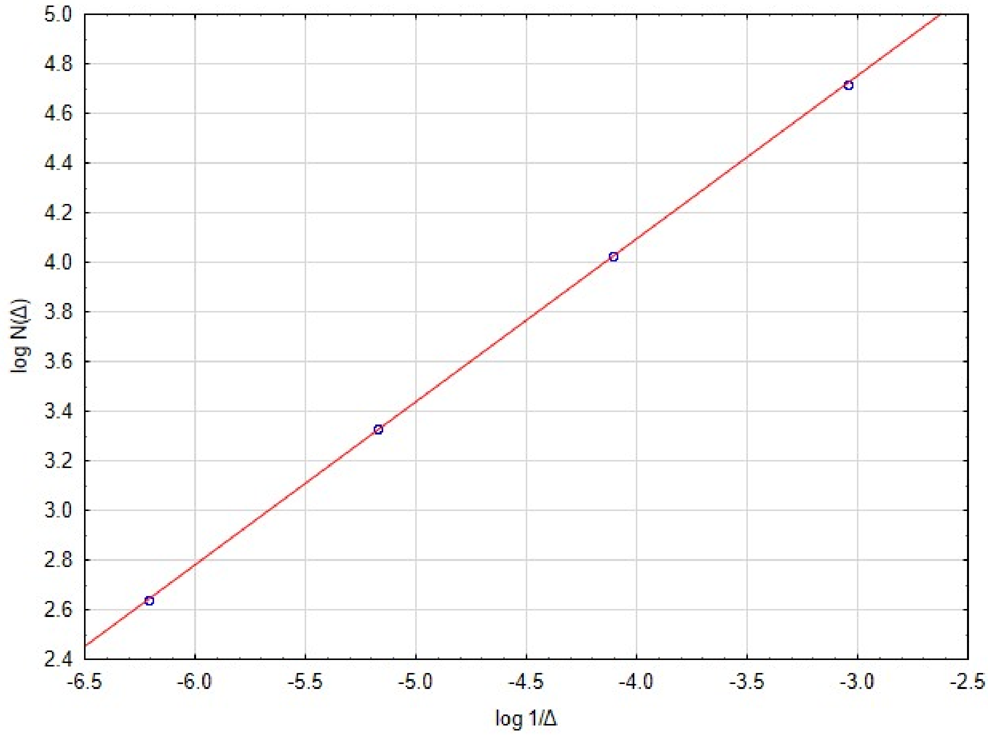


Fig. 1. Hripsime: graph of the dependence of $\log N(\Delta)$ on $\log(1/\Delta)$.

4.2 Zvartnots

The Zvartnots Cathedral was founded by Catholicos Nerses III in the middle of the 7th century, not far from Vagharshapat (Echmiadzin) in the place where, according to legend, Gregory the Illuminator and the king of Armenia Trdat met. This majestic temple is a tetraconch (a central-domed structure with a plan in the form of a cross with rounded ends).

Appendix 2 contains fragments of the process of calculating the fractal dimension of the facade and plan of the Zvartnots temple. The results of the calculations are summarized in Table 2.

Table 2. Fractal dimension of the Zvartnots Cathedral facade and plan.

Calculation of fractal dimension between:		fractal dimension	
large grid size	small grid size	facade	plan
200	100	1.64	1.67
100	50	1.54	1.57
50	25	1.48	1.49
25	12.5	1.47	1.43
general fractal dimension		1.533	1.540

From the obtained data, it follows that the Zvartnots temple has an average fractal dimension of 1.533. The calculations also show that the standard deviation of these data from the average is 0.008. Regarding the architectural plan, the following estimates were obtained: the average fractal dimension is 1.54 with a standard deviation of 0.104. The correlation between the fractal dimensions of the facade and the plan is 0.974.

Fig. 2 shows a graph of the dependence of $\log N(\Delta)$ on $-\log\Delta$ for the facade of the temple, which is a linear regression constructed using the obtained values of the fractal dimension.

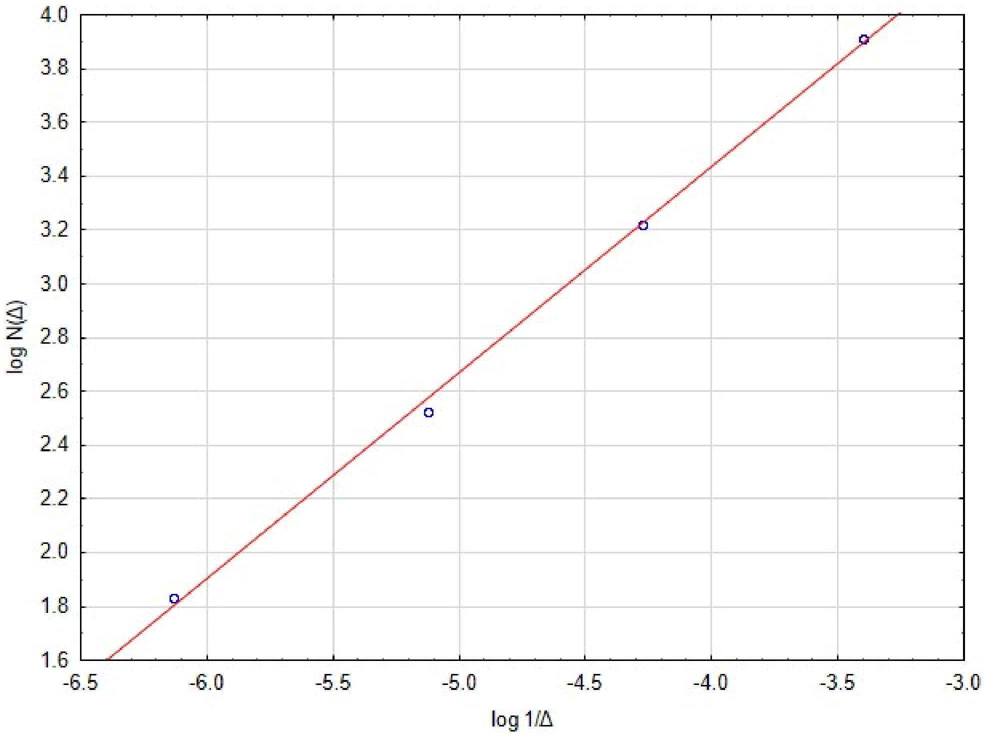


Fig. 2. Zvartnots: graph of the dependence of $\log N(\Delta)$ on $\log(1/\Delta)$.

4.3 Cathedral of the Holy Virgin in Ani

The Ani Cathedral is the pinnacle of Armenian architecture of the 9th-11th centuries. It is a prototype of Gothic architecture. Its architectural forms are similar to European Gothic.

Regarding Gothic, we note that there is a very reasonable assumption that the first object where Gothic principles were applied was not the Cathedral of Saint-Denis (a suburb of Paris), but the Cathedral of the Holy Virgin in Ani. The interior of this temple clearly contains such architectural compositions as elongated pointed arches, bunches of columns with ribbed vaults. These compositions were developed in Gothic architecture, which was widespread in Western Europe.

In his major work [19], Professor of the University of Vienna J. Strzygowski writes: "Consequently, it remains to be recognized that the Armenians built in the Gothic style approximately 150 years earlier than was the case in Europe".

Appendix 3 contains fragments of the process of calculating the fractal dimension of the facade and plan of the Ani Cathedral. The results of the calculations are summarized in Table 3.

Table 3. Fractal dimension of the Ani Cathedral facade and plan.

Calculation of fractal dimension between:		fractal dimension	
large grid size	small grid size	facade	plan
200	100	1.56	1.48
100	50	1.53	1.50
50	25	1.56	1.43
25	12.5	1.5	1.13
general fractal dimension		1.537	1.385

From the obtained data, it follows that the Ani Cathedral has an average fractal dimension of 1.537. The calculations also show that the standard deviation of these data from the average is 0.029. Regarding the architectural plan, the following estimates were obtained: the average fractal dimension is 1.385 with a standard deviation of 0.172. The correlation between the fractal dimensions of the facade and the plan is 0.797.

Fig. 3 shows a graph of the dependence of $\log N(\Delta)$ on $-\log \Delta$ for the facade of the temple, which is a linear regression constructed using the obtained values of the fractal dimension.

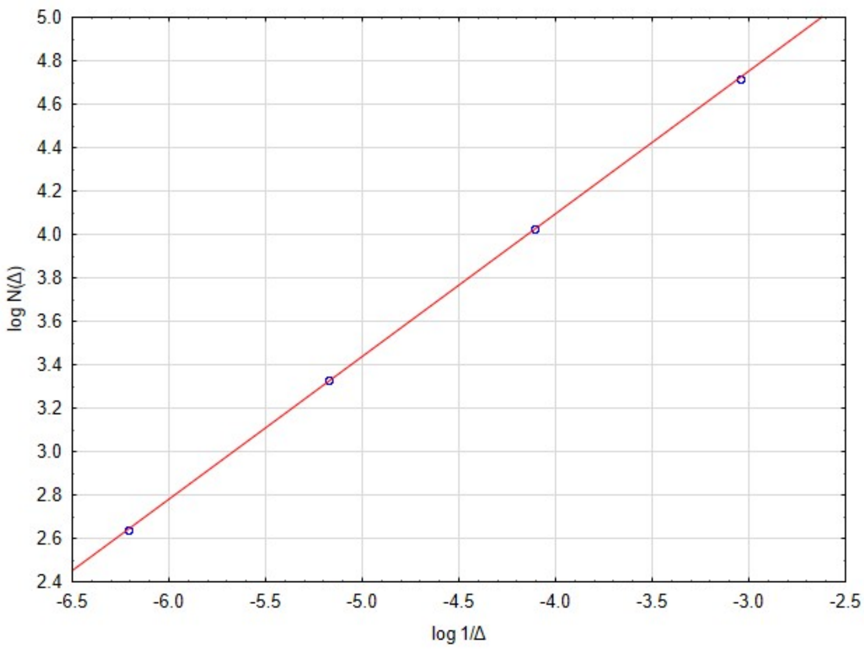


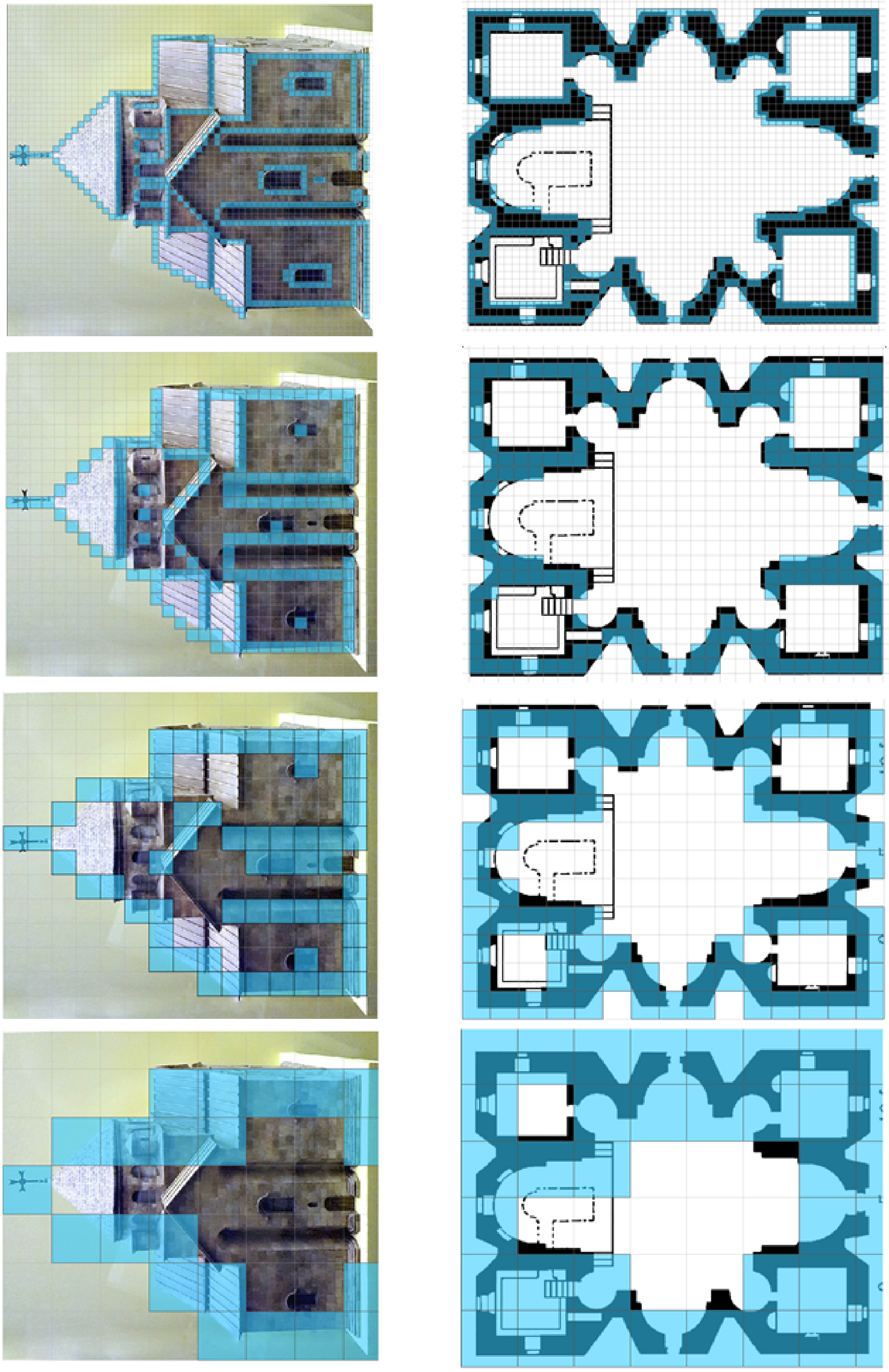
Fig. 3. Cathedral in Ani: graph of the dependence of $\log N(\Delta)$ on $\log(1/\Delta)$.

5. Conclusion

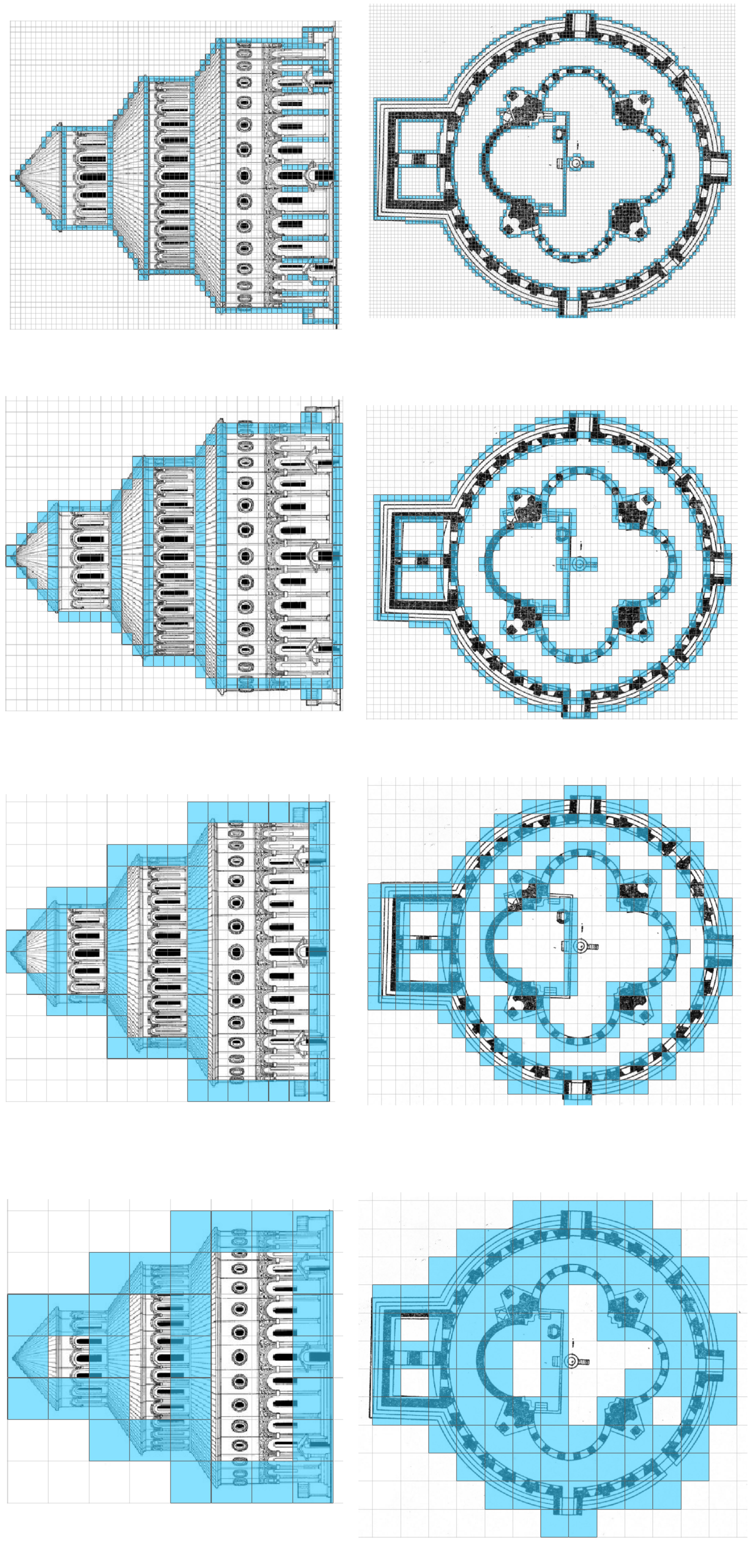
Fractal analysis of the examined Armenian churches showed a high level of consistency between subjective and objective assessments of their aesthetic appeal.

Appendix

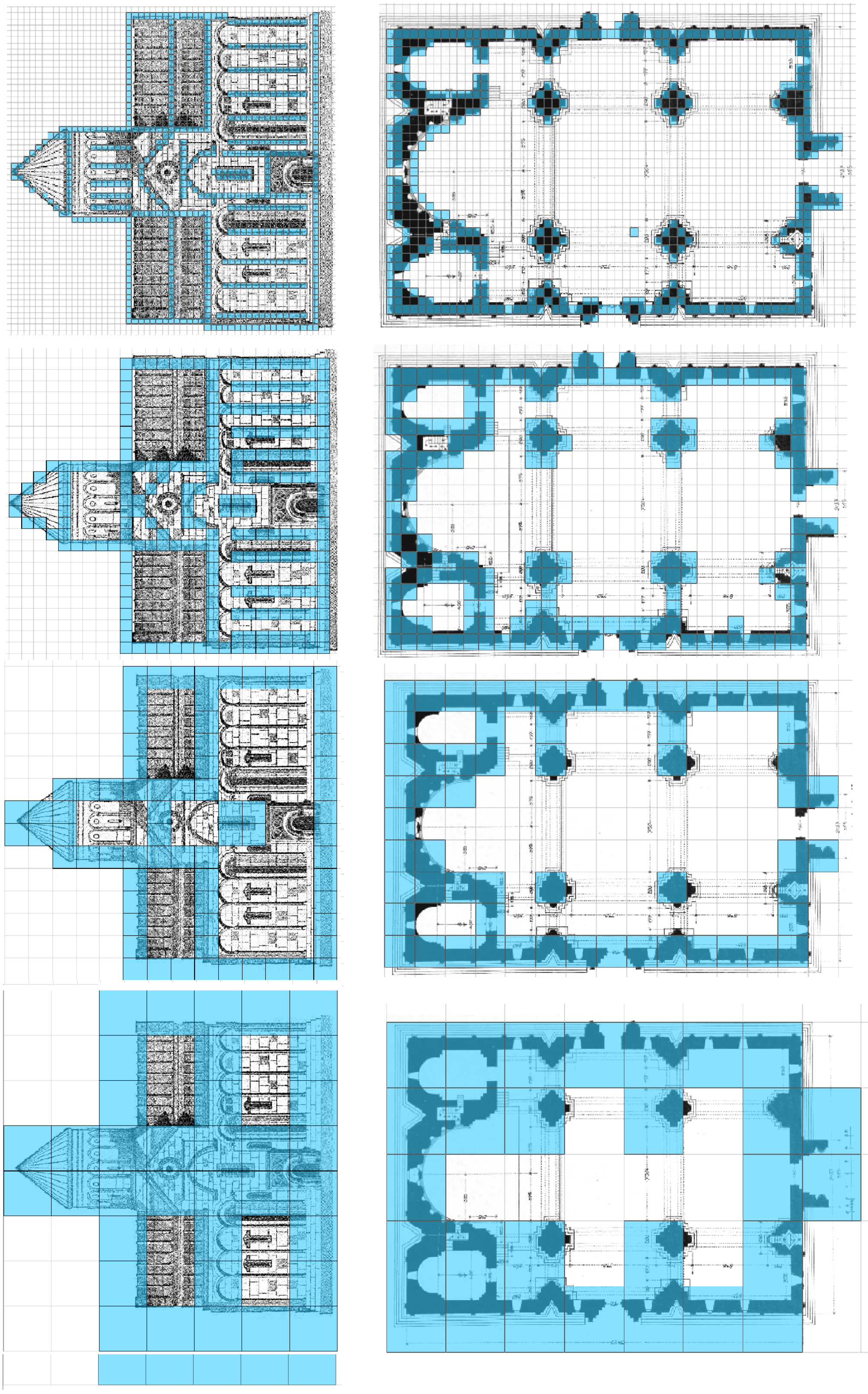
Appendix 1. Calculation of the fractal dimension of the facade and plan of the Hripsime temple.



Appendix 2. Calculation of the fractal dimension of the facade and plan of the Zvartnots temple.



Appendix 3. Calculation of the fractal dimension of the facade and plan of the Ani Cathedral.



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Ֆրակտալ երկրաչափություն և հին հայկական ճարտարապետական հուշարձանների գեղագիտական գրավչության քանակական գնահատում

Արեն Ա. Նահապետյան

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Ամփոփում

Ներկայացվել է միջնադարյան Հայաստանի ամենակարևոր ճարտարապետական հուշարձանների (տաճարների) ֆրակտալ վերլուծություն: Ստացվել են դրանց գեղարվեստական գրավչության քանակական (օբյեկտիվ) գնահատականներ: Այս արդյունքները հաստատում են տվյալ հուշարձանների՝ որպես հայկական ճարտարապետակյան գլուխգործոցների ընդունված (սուբյեկտիվ) գնահատականը: Ստացված ֆրակտալ տվյալների հիման վրա կատարվել են վիճակագրական եզրակացություններ դիտարկվող հուշարձանների հատակագծերի և ճակատների ճարտարապետական համապատասխանության բարձր աստիճանը:

Բանալի բառեր՝ ֆրակտալ, ֆրակտալ չափականություն, ճարտարապետություն, հոգեբանություն:

Фрактальная геометрия и количественная оценка эстетической привлекательности памятников древнеармянской архитектурыр

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Аннотация

Дан фрактальный анализ важнейших архитектурных памятников (храмов) средневековой Армении. Получены количественные (объективные) оценки их художественной привлекательности. Эти результаты подтверждают общепринятую (субъективную) оценку этих храмов как шедевров армянской архитектуры. На основе полученных фрактальных данных сделаны статистические выводы о высокой степени архитектурной совместимости планов и фасадов рассматриваемых памятников.

Ключевые слова: фрактал, фрактальная размерность, архитектура, психология.