

UDC 519.714

RDNF Oriented Analytics to Random Boolean Functions

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Abstract

Dominant areas of computer science and computation systems are intensively linked to the hypercube-related studies and interpretations. This article presents some transformations and analytics for some example algorithms and Boolean domain problems. Our focus is on the methodology of complexity evaluation and integration of several types of postulations concerning special hypercube structures. Our primary goal is to demonstrate the usual formulas and analytics in this area, giving the necessary set of common formulas often used for complexity estimations and approximations. The basic example under considered is the Boolean minimization problem, in terms of the average complexity of the so-called reduced disjunctive normal form (also referred to as complete, prime irredundant, or Blake canonical form). In fact, combinatorial counterparts of the disjunctive normal form complexities are investigated in terms of sets of their maximal intervals. The results obtained compose the basis of logical separation classification algorithmic technology of pattern recognition. In fact, these considerations are not only general tools of minimization investigations of Boolean functions, but they also prove useful structures, models, and analytics for constraint logic programming, machine learning, decision policy optimization and other domains of computer science.

Keywords: Boolean function, Hypercube, Complexity, Asymptotic, Reduced disjunctive normal form.

Article info: Received 14 February 2023; sent for review 10 March 2023; accepted 11 April 2023.

1. Hypercube and Related Structures

The metric theory of Boolean functions (BF) [1], [2] arose in the 70's, in parallel with the emergence of broader design and implementation ideas for mechanical and electronic computation devices. It was then that it turned out that the system of binary representation of numbers is the most optimal, both from the point of view of the algorithmic implementation of arithmetic calculations and also from the point of view of developing physical carriers of performing these calculations [3]. BF – functions with only binary variables, and also

with values in the domain $\{0, 1\}$, although simple among the other similar mathematical concepts, they are quite complex in solving problems associated with their transformations and optimization. The metric theory of Boolean functions provides the necessary knowledge for coding, transforming and implementing binary functions. Although the way to minimal BF representations are and remains difficult, a rather complete picture of the main forms of function representation of functions has been obtained, and the basic role here takes the concept of disjunctive normal forms. Successive steps of several transformations of functions are found to achieve minimal forms as a chain from the table or formula representation to the reduced d.n.f., then to the deadlock forms and finally – the minimal structures. The accompanying structures and bottlenecks of achieving acceptable optimization are investigated intensively [1], [4]–[7]. Here we will not cover the whole theory but will pay attention to one fundamental construction, – to the concept of reduced disjunctive normal forms (r.d.n.f.) of Boolean functions. R.d.n.f. is the collection of all minimal conjunctions and geometrically – the system of all maximum intervals/sub-cubes of functions. These forms are a universal concept, and they also arise in problems such as circuit design from set of functional elements (logical part of chip design), in the theory of pattern recognition (logic separation algorithm, and generation of logical regularities) [8]–[11], in biological models of heredity and mutations (phylogeny, parsimony) [12, 13], etc. Turning to the complexity characterization of structures associated with the reduced disjunctive normal form, where two types are usually considered: the largest and most typical characteristics, we will focus on the second component. In a concise survey of the domain, the initial studies of [5], [14], and [15], should be mentioned, that give the formulas of average numbers of maximal intervals in Boolean functions. [16], [17] extended these results to the case of partially defined Boolean functions. An alternative track of papers in these topics includes the articles [18], [19], [20]. Current research on the topics of BF and complexities might be demonstrated through the papers [21]–[26]. Methodologically, in studies in the area of BF, it should be taken into account that the function determination domain, as well as the number of functions itself, are finite, depending on the number of the variables – the dimensionality. So, considering the parameter $\pi(f)$ over the functions, we get the split of these functions into finite classes by the values of this parameter. These are the rates and intensity of the accepted values of the parameter $\pi(f)$. In some cases, it is convenient to refer to these valuations as probabilistic distributions, which is not obligatorily but is convenient in some contexts. In this concern, there appears a link to the model of Random Boolean functions and the combinatorial theories initiated by A. Renyi and P. Erdos [27], [28].

1.1 Concepts and Definitions in the Binary Domain

Elementary conjunction, Direction. Let $\tilde{\alpha}$ and $\tilde{\beta}$ – be arbitrary vertices of the n -dimensional unite cube. And let $j_i, i = 1, 2, \dots, r$ be all coordinates, those where $\alpha_{j_i} = \beta_{j_i}$. Consider the formula

$$\mathcal{K}(x_1, x_2, \dots, x_n) = \bigwedge_{i=1}^r x_{j_i}^{\sigma_{j_i}},$$

with $\sigma_{j_i} = \alpha_{j_i}, i = 1, 2, \dots, r$. We say that K is an elementary conjunction stretched on the pair of vertices $\tilde{\alpha}$ and $\tilde{\beta}$ of the n -dimensional unit cube E_n . The number of literals in \mathcal{K} is the rank of \mathcal{K} . The geometrical counterpart of \mathcal{K} is a sub-cube defined as follows. Assign 0 values to all but j_1, j_2, \dots, j_r coordinates and denote this vertex by v_0 . Similarly, assign

these coordinates by the value 1, obtaining the vertex v_1 . These are the minimal and maximal vertices that belong to \mathcal{K} , and they determine a unique sub-cube of all truth vertices of \mathcal{K} . $n - r$, the number of variable coordinates of \mathcal{K} is the size of its sub-cube.

Let $\lambda = \{j_1, j_2, \dots, j_r\}$ be a collection of r indices drawn up of variables x_1, x_2, \dots, x_n , and let $\bar{\lambda}$ be the complementary to the λ set of indices. Conjunctions of the form $\bigwedge_{i=1}^r x_{j_i}^{\sigma_{j_i}}$ and the corresponding intervals will be called conjunctions and intervals of the direction λ . For a fixed r there are C_n^r different directions, and each of them is determined by the appropriate selection of an r subset $\{j_1, j_2, \dots, j_r\}$ of the set $\{1, 2, \dots, n\}$. The individual interval in the direction $\{j_1, j_2, \dots, j_r\}$ appears in result of assigning the values $\sigma_1, \sigma_2, \dots, \sigma_r$ to the variables $x_{j_1}, x_{j_2}, \dots, x_{j_r}$.

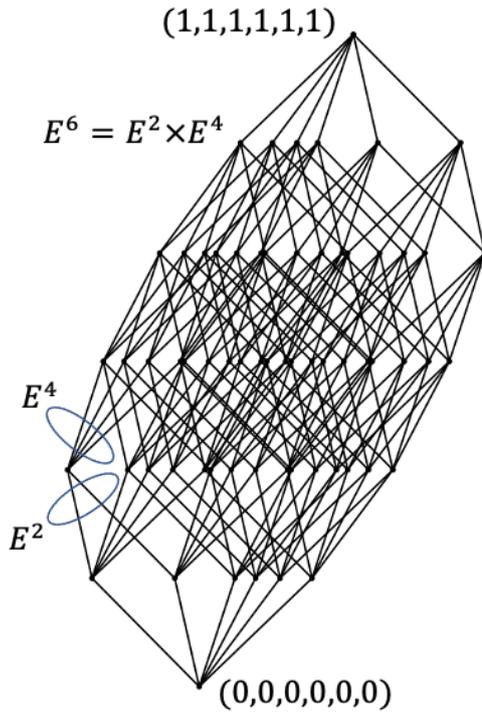


Fig. 1. Geometry of hypercube.

This also means that there are 2^{n-r} conjunctions and intervals in one of the r -directions. The collection $\bar{\lambda}$ of indices defines another set of directions.

Let \mathcal{F} be an arbitrary logical formula and $\mathcal{M} \subseteq B^n$. We say that \mathcal{F} absorbs or covers \mathcal{M} if on each tuple $\tilde{\alpha} \in \mathcal{M}$ the formula \mathcal{F} accepts the unite (true) value.

Let $\tilde{\alpha} \in E^n$ be an arbitrary vertex. Call the value $|\tilde{\alpha}| = \sum_{i=1}^n \alpha_i$ the module or the weight of $\tilde{\alpha}$. The set of all vertices $\tilde{\beta} \in E^n$, with $\rho(\tilde{\alpha}, \tilde{\beta}) = |\tilde{\alpha} \oplus \tilde{\beta}| = k$, call the k -the layer of E^n in relation to the vertex $\tilde{\alpha}$ (\oplus - mentions *mod2* summation).

Intervals \mathcal{N}_{K^1} and \mathcal{N}_{K^2} ,

$$K^1(x_1, x_2, \dots, x_n) = \bigwedge_{i=1}^r x_{j_i}^{\sigma_{j_i}^1} \text{ and } K^2(x_1, x_2, \dots, x_n) = \bigwedge_{i=1}^r x_{j_i}^{\sigma_{j_i}^2}$$

of the same size and the same direction we call neighbors if $\rho(\tilde{\sigma}^1, \tilde{\sigma}^2) = 1$, where ρ - be the Hamming distance, $\rho(\tilde{\sigma}^1, \tilde{\sigma}^2) = \sum_{i=1}^r |\sigma_{j_i}^1 - \sigma_{j_i}^2|$. Let then j_{i_0} is the number of that unique coordinate for which $\sigma_{j_{i_0}}^1 \neq \sigma_{j_{i_0}}^2$. Then we say that the conjunctions K^1 and K^2 (or the pair of neighbor intervals corresponding to them) joined by the coordinate $x_{j_{i_0}}$, and, as a result,

a new conjunction (interval) appears:

$$\bigwedge_{i \neq i_0, i=1}^r x_{j_i}^{\sigma_{j_i}}.$$

Partition the variable set x_1, x_2, \dots, x_n in an arbitrary manner into two nonempty groups: $x_{i_1}, x_{i_2}, \dots, x_k$ as the first group, and $x_{i_{k+1}}, x_{i_{k+2}}, \dots, x_{i_n}$ as the second. Then, the n -dimensional unit cube E_n may be represented as the Cartesian multiplication $B^k \times B^{n-k}$ of two sub-cubes: B^k and B^{n-k} generated correspondingly by the sets of variables $x_{i_1}, x_{i_2}, \dots, x_{i_k}$ and $x_{i_{k+1}}, x_{i_{k+2}}, \dots, x_{i_n}$. Let us enumerate the vertices of B^{n-k} by the layers relative to the vertex $\tilde{0}$ of B^{n-k} . Enumeration among the vertices of a particular layer is arbitrary, but the first group that is enumerated by low numbers is layer zero, then the first layer, and so on. Additional ordering among layer vertices may use lexicographic order, binary value based order, etc.

Consider an arbitrary k -dimensional sub-cube B^k of E^n , the first k -dimensional interval B_1^k in the direction of B^k . List the neighbor intervals to the considered one, $B_1^k, B_2^k, B_3^k, \dots, B_{n-k+1}^k$. Let f be an arbitrary (partially defined) Boolean function that satisfies the following conditions:

- $\alpha)$ B_1^k doesn't contain zero value vertices of f : $(\forall \tilde{\alpha} \in B_1^k, f(\tilde{\alpha}) \neq 0)$,
- $\beta)$ Each of the neighbor with B_1^k interval contains at least one 'unit' value vertex f : $(\forall j, j = 2, 3, \dots, n - k + 1 \exists \tilde{\alpha} \in B_j^k, f(\tilde{\alpha}) = 1)$,
- $\gamma)$ B_1^k contains at least one 'unit' vertex of f : $(\exists \tilde{\alpha} \in B_1^k, f(\tilde{\alpha}) = 1)$.

In conditions $\alpha), \beta), \gamma)$, we say that B_1^k is a maximal interval of the function f . d.n.f., composed of all elementary conjunctions, corresponding to maximal intervals of function f is named the reduced disjunctive normal form of f . The number of disjunctive members of this formula is considered as its complexity. Denoting by $r_k(f)$ the number of all maximal k -intervals of the function f we get the formula of complexity of the reduced disjunctive normal form of f :

$$\sum_{k=0}^n r_k(f).$$

2. On the Maximum Number of k -Dimensional Maximal Intervals of RBF

Consider the class $P_2(n)$ of all Boolean functions of n variables x_1, x_2, \dots, x_n . Let $p, 0 < p < 1$ be a fixed number, and F_p – the probability distribution on $P_2(n)$, generated in the following way. The function $f \in P_2(n)$ is induced as a result of a randomized experiment, where the values of the function on vertices of E^n are derived randomly. The value 1 appears with a probability p and the 0 value – with a complementary probability $1 - p$. The vertices of E^n take part in this experiment independently of each other, and the probabilistic distribution F_p over the set of Boolean functions is generated in this way. The probability of an individual Boolean function f under the distribution F_p depends on the balance between the 0 and 1 values of the function f (the volumes of the sets $\mathcal{N}_\{1\}$ and $E^n - \mathcal{N}_\{1\}$). For $f \in P_2(n)$, this probability is equal to $p^{|\mathcal{N}_\{1\}|} (1 - p)^{2^n - |\mathcal{N}_\{1\}|}$. When $p = 1/2$ this probability is simply $1/2^{2^n}$ and the corresponding distribution becomes the uniform distribution over

the $P_2(n)$. We introduce the notation $r_k(f)$ for the number of k -dimensional maximal intervals of the function $f \in P_2(n)$. And let $r_k(n, p)$ be the average value of the number of k -dimensional maximal intervals of functions $f \in P_2(n)$ under the distribution F_p . It is easy to make sure, that

$$r_k(n, p) = \sum_{f \in P_2(n)} F_p(f) * r_k(f) \quad (1)$$

The number $r_k(n, p)$ in the expression (1) is given by its definition as a sum over all functions of $f \in P_2(n)$, counting all their k -dimensional maximal intervals and taking into account the probabilities of f in the distribution F_p .

Further evidence of these constructions is provided by the following scheme:

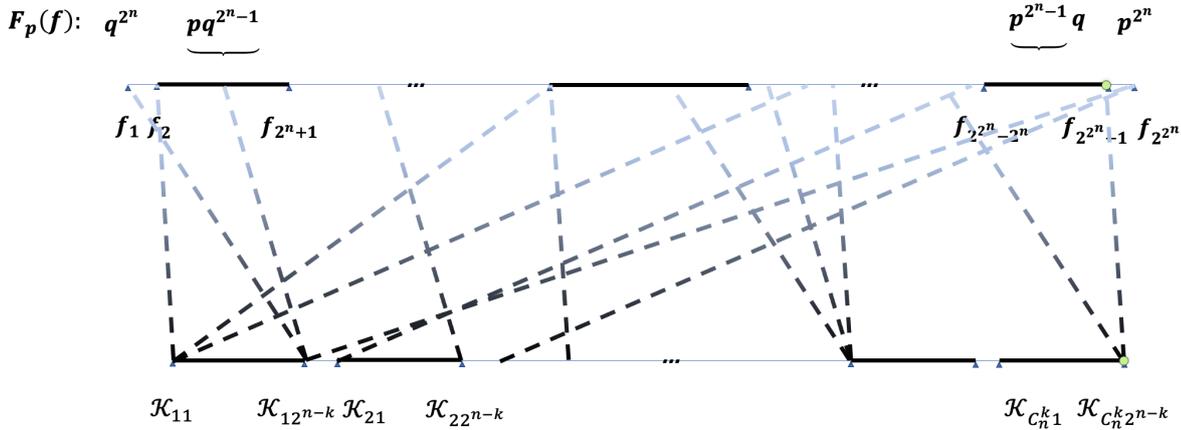


Fig. 2. This figure presents the bipartite graph of functions and k -dimensional maximal intervals. Upper line functions are placed in order of the number of their "true" values, from 0 to 2^k . Different functions include different numbers of k -dimensional maximal intervals and have different probabilities under the distribution F_p . Instead, each interval presented in the bottom line is connected to the same number of functions. This is because the sizes of intervals is the same. The order of intervals is by groups of intervals, that belong to the same direction. Numeration inside the functions with the same number of "ones" and inside the groups of intervals of the same direction is arbitrary.

Following [5], we change the order of counting in 1, first considering all k -dimensional intervals in E^n . We relay two events to these intervals: the one, about their maximality, and then the second, about the set of functions that accept the first event about maximality. In this regard, it is also convenient to split the E^n in parts: the current k -dimensional interval \mathcal{K} and its all $n - k$ neighboring k -dimensional intervals $\mathcal{K}_1, \mathcal{K}_2, \dots, \mathcal{K}_{n-k}$. This part, the current interval and its neighbors, covers an area \mathcal{E}_1 of $2^k(n - k + 1)$ vertices of E^n . And the second part that we consider, consists of the complementary area \mathcal{E}_2 to \mathcal{E}_1 up to E^n . The probability of maximality of \mathcal{K} for the function f becomes the product of probability of maximality of \mathcal{K} together with the conditional probability of f when \mathcal{K} is given to be maximal. The first probability equals $p^{2^k}(1 - p^{2^k})^{n-k}$. The first and second parts consist of events, and their sums of probabilities are equal to 1 as a probabilistic distribution. Now, when we sum up the mentioned conditional probabilities with all f , we get the probability 1, and the final probability of maximality of \mathcal{K} , under the conditions of F_p , becomes $p^{2^k}(1 - p^{2^k})^{n-k}$. It reminds us to take this probability for all k -dimensional intervals, obtaining the following equivalent form for (1),

$$r_k(n, p) = C_n^k 2^{n-k} p^{2^k} (1 - p^{2^k})^{n-k}. \quad (2)$$

Theorem 1. $r_k(n, p)$ is a concave function of the parameter k in the interval $[0, n]$.

It is important to know the behavior of the function $r_k(n, p)$ defined on the interval $[1, n]$. Initially, it is useful to calculate the values of the function at the boundary points of the domain of definition: $k = 0, 1, \dots, n-1, n$. We give these values both for the arbitrary p and the value $1/2$.

Table 1: Values of $r_k(n, p)$ on boundary points, such as $k = 0, 1, \dots, n-1, n$.

Boundary point values of $r_k(n, p)$		
Dimension k of maximal interval	$r_k(n, p)$	$r_k(n, 1/2)$
$k = 0$	$2^n p(1-p)^n$	$1/2$
$k = 1$	$n2^{n-1}p^2(1-p^2)^{n-1}$	$(n/4)(3/2)^{n-1}$
...
$k = n-1$	$n2^{n-1}p^{2^{n-1}}(1-p^{2^{n-1}})$	$n2^{n-1}(1-1/2^{2^{n-1}})/2^{2^{n-1}}$
$k = n$	p^{2^n}	$1/2^{2^n}$

As we can see, both the left and right boundary point values of the interval $(0, n)$ are small, but there is a noticeable increase from left to right at the left end, and a decrease from left to right at the right end. To get a complete picture of the behavior, consider a number of special intermediate point values of the function at:

$$k_1 = \log \frac{1}{-\log p}, \quad k_0 = \log \frac{\log n}{-\log p}, \quad \text{and} \quad k_2 = \log \frac{n}{-\log p}.$$

The technical element of choosing of these values is in simple evaluation of sub-formula $E_k = 2^{2^k}$, which is an important functional part of the 1. Substituting k_1 , k_0 , and k_2 into E_k we get:

$$E_{k_1} = 1/2, \quad E_{k_0} = 1/n, \quad E_{k_2} = 1/2^n. \quad (3)$$

Let us start the proof of postulations 1-3. For this, conduct a preliminary analysis of the expression (2) for $r_k(n, p)$. Consider an arbitrary integer value function $k(n)$ that obeys the restriction $0 \leq k(n) \leq n$, and substitute it into the expression 2. We are interested in the behaviour of the received function $r_{k(n)}(n, p)$ depending on the parameter $k(n)$ as $n \rightarrow \infty$.

First let's make sure that with the increase of k the expression $r_k(n, p)$ increases monotonically by the $k \leq [k_0]$, and then it decreases, when $]k_0[\leq k$. By doing this we compose the relation

$$R_k = \frac{r_{k+1}(n, p)}{r_k(n, p)} = \frac{(n-k)p^{2^k}(1+p^{2^k})^{n-k}}{2(k+1)(1-p^{2^{k+1}})}. \quad (4)$$

This expression can be considered for an arbitrary (not only for the integer) assignment to the parameter k . We will follow by checking if this function is concave in the interval $0 < k < n$ for large n . The direct way of this is to derive the expression of the fraction

R_k and treat it for a possible constant/zero value of it. In such consideration, the most important role takes the part $A_k = (n - k)p^{2^k}$ of the base expression 4. Substituting k_0 into A_k we obtain that $(n - k_0)p^{2^{k_0}} = (n - k_0)p^{\left(\frac{\log n}{-\log p}\right)} = (n - k_0)2^{\log p \left(\frac{-\log n}{-\log p}\right)} = \frac{n - k_0}{n}$, which is converging to 1 as $n \rightarrow \infty$. With the help of formulas in Section 3. we see that the part $B_k = (1 + p^{2^k})^{n-k}$ of (4) is limited at the point k_0 : (6) gives $(1 + p^{2^{k_0}})^n \rightarrow e$ as $n \rightarrow \infty$, so that $(1 + p^{2^{k_0}})^{n-k_0}$ also tends to e . Compose the fraction B_{k+1}/B_k in the following form:

$$B_{k+1}/B_k = \frac{(1 + p^{2^k} p^{2^k})^{n-k-1}}{(1 + p^{2^k})^{n-k}} = \frac{\left(\frac{1+p^{2^k} p^{2^k}}{1+p^{2^k}}\right)^{n-k}}{1 + p^{2^k} p^{2^k}} \quad (5)$$

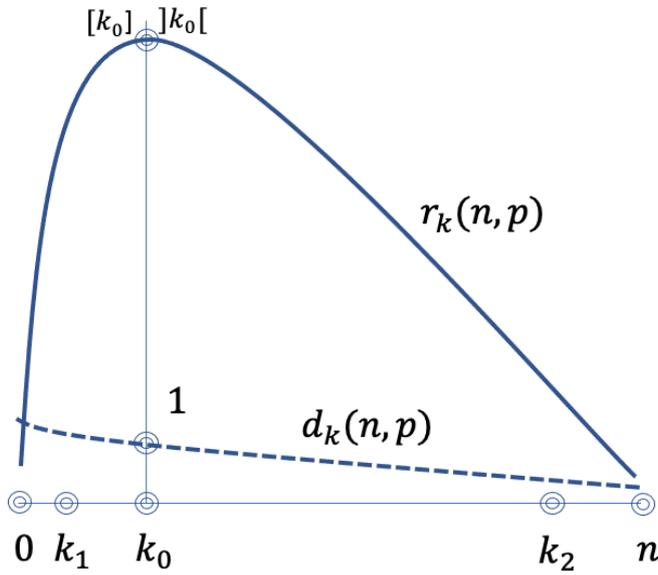


Fig. 3. Differential of growing $r_k(n, p)$.

Note that the fraction $\frac{1+p^{2^k} p^{2^k}}{1+p^{2^k}}$ is less than 1, so its $n - k$ degree is also less than 1. And the denominator of (5) is greater than 1 so that, finally, the expression (5) is less than 1 for all k , which means a monotonic decrease of the expression R_k in (5). In general, as k increases, all the factors of (4), other than B_k , decrease monotonically and, besides this, as $n \rightarrow \infty$, this expression tends to zero at the point k_0 and grows infinitely when $k = k_0 - 1$. Finally, we receive that with increasing k , for the beginning, $i_k(n, p)$ increases, achieving its maximal value at the point $[k_0]$ or $]k_0[$, and, then, it decreases.

3. On the Dependency of Number of k -Dimensional Maximal Intervals on k

Consider the parameter $k_2 = \log \frac{n}{-\log p}$. Since $0 < p < 1$, we have $k_2 = \log n + c$, where c represents an absolute constant determined by the fixed value of p . We intend to obtain an asymptotic formula for $i_k(n, p)$ by the $n \rightarrow \infty$ for the values of k of the form $k_2 + \text{const}$. We make use of the following expressions $C_n^k \sim \frac{n^k}{k!}$, $(1 - p^{2^k}) \sim 1$, and $n! \sim n^n e^{-n} \sqrt{2\pi n}$ as $n \rightarrow \infty$, which are based on the formulas

1. If $0 \leq x \leq 1$ and $0 \leq y$, then

$$\exp\left(x\left(1 - \frac{x}{2}\right)y\right) \leq (1 + x)^y \leq \exp(xy). \quad (6)$$

2. If $0 \leq x \leq 1$ and $0 \leq y$, then

$$(1-x)^y \leq \exp(-xy); \text{ and} \quad (7)$$

$\exp(-x(1-x)y) \leq (1-x)^y$, when additionally $0 \leq x \leq 1/2$.

3. If x and y be natural numbers, and $x \leq y$, then

$$\left(1 - \frac{x}{y}\right)^{\frac{x-1}{2}} \leq \prod_{i=1}^{x-1} \left(1 - \frac{i}{y}\right) \leq \left(1 - \frac{x}{2y}\right)^{x-1}. \quad (8)$$

and are valid for the mentioned values of the parameter k , and for this reason

$$i_k(n, p) \sim \frac{n^k e^k 2^{n-k} p^{2^k}}{k^k \sqrt{2\pi k}} = \tilde{i}_k(n, p). \quad (9)$$

Theorem 2. *The probability, that functions of the class $P_2(n)$ under the distribution F_p have maximal intervals of sizes k , $k < [k_1]$ or $k > [k_2]$, where $k_1 = \log_{\frac{1}{-\log p}}$ and $k_2 = \log_{-\log p} n$ tends to zero with $n \rightarrow \infty$.*

On the right side of (9) we have expression, that depends on the continuous argument k , and which is equivalent to the expression $i_k(n, p)$ for the integer values of the parameter k , of the form $k_2 + \text{const}$. In the mentioned area, $\tilde{i}_k(n, p)$ decreases monotonically with the increase of k , $\tilde{i}_{k_2}(n, p)$ tends to infinity, and $\tilde{i}_{k_2+1}(n, p)$ tends to zero, when $n \rightarrow \infty$, so that $i_k(n, p) \rightarrow 0$, for values $k >]k_2[$ and $i_k(n, p) \rightarrow \infty$ for values $k_0 \leq k \leq [k_2]$, by $n \rightarrow \infty$. Let us also denote, that we do not insist that $i_{]k_2[}(n, p)$ as $n \rightarrow \infty$ converges to any appropriate value.

In what follows, we will use the first Chebyshev inequality (1). The first inequality lets formulate an extension of a postulation from [29] for the case of the probability distribution F_p . Actually, if to consider the expression $i_k(f)$, as a parameter of $\pi(f)$ then for the values $k >]k_2[$ $i_k(n, p) \rightarrow 0$ by $n \rightarrow \infty$, and taking into the force the first inequality for the arbitrary $\epsilon(n) \geq 0$ $P(i_k(f) \geq \epsilon(n)) \rightarrow 0$ when $n \rightarrow \infty$.

A similar situation takes place in the region of small values of the parameter k . For the value $k = k_1$ and $p = 1/2$ by the (3) $p^{2^{k_1}} = 1/2$ and $r_{k_1}(n, p) \rightarrow \infty$ as $n \rightarrow \infty$. For $p > 1/2$, already for the value $k_1 - 1$, we observe that $r_{k_1-1}(n, p) \rightarrow 0$ as $n \rightarrow \infty$. This is just because $\frac{2^{n-k_1+1}}{1-p^{2^{k_1-1}}}$ is a decreasing exponent, which together with C_n^k tends to 0.

4. Conclusion

This article has two goals: first, it considers the set of formulas needed to analyze the complexity of structures associated with a multidimensional unit cube, providing the necessary transformations and approximations for these formulas. Further, the paper considers a typical study for this field using these formulas. The problem under consideration estimates the complexity of the reduced disjunctive normal form of Boolean functions on average, or, what is the same, for almost the entire class of functions.

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Պատահական բուլյան ֆունկցիաների ԿԴՆՉ կողմնորոշված վերլուծություն

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Ամփոփում

Այս հոդվածն ունի երկու նպատակ, նախ այն քննարկում է բանաձևերի մի շարք, որոնք անհրաժեշտ են բազմաչափ միավոր խորանարդի հետ կապված կառուցվածքների բարդությունը վերլուծելու համար՝ ապահովելով անհրաժեշտ փոխակերպումներ և մոտարկումներ այս բանաձևերի համար: Ավելին, հոդվածը քննարկում է այս ոլորտի համար որոշ բնորոշ ուսումնասիրություն՝ օգտագործելով այս բանաձևերը: Քննարկվող ընթացակարգը գնահատում է բուլյան ֆունկցիաների կրճատված դիզյունկտիվ նորմալ ձևի բարդությունը միջինում կամ, որ նույնն է, դասի գրեթե բոլոր ֆունկցիաների համար:

Բանալի բառեր Բուլյան ֆունկցիա, բազմաչափ միավոր խորանարդ, բարդություն, ասիմպտոտիկա, կրճատված դիզյունկտիվ նորմալ ձև:

Аналитика ориентированная на СДНФ случайных булевых функций

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Аннотация

Данная статья преследует две цели: во-первых, в ней рассматривается набор формул, необходимых для анализа сложности структур, связанных с многомерным единичным кубом, предоставляя необходимые преобразования и аппроксимации для этих формул. Далее, в статье рассматривается типичное исследование для данной области с использованием этих формул. Рассматриваемая проблема оценивает сложность сокращенной дизъюнктивной нормальной формы булевых функций в среднем, или, что то же самое, почти для всего класса функций.

Ключевые слова: булева функция, многомерный единичный куб, сложность, асимптотика, сокращенная дизъюнктивная нормальная форма.