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An Approach to General Form Recursive Equation Solutions

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Research in finding solutions of functional equations have a long history. In 1791 A. Legendre [4] posed the problem of finding the continuous solutions of the equation

$$f(x+y) = f(x) + f(y) \tag{1}$$

It was dealt with as Legendre and Gauss, but its solution, as well as other similar equations, but managed to get by A. Cauchy [1]. Later the functional equations were investigated by many authors.

The present author began research in search of computable solutions to general form recursive equations (GFRE) (see [5], [6]):

$$\begin{array}{l} F_1[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m) \simeq G_1[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m); \\ F_2[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m) \simeq G_2[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m); \\ \hfill ; \\ F_k[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m) \simeq G_k[f_1, f_2, \ldots, f_n](x_1, x_2, \ldots, x_m); \end{array} \tag{2}$$

where x_i are numeric variables,f_i- are variables denoting the desired functions, F_i, G_i - recursive (computable) functionals. In [5] it was shown that the problem of the existence of certain solutions to the systems of the type (2) is Σ_3^0- complete if looking for partial solutions and is Σ_1^1-complete if looking for total solutions. It is evident, that any system of differential, integral or even mixed type of implicit equations, rewritten in a form suitable for numerical computation, brings to form of a GFRE. Hence it is natural to face to problems concerning the search for solutions to these equations. P. Collins and D. Bauman [2] examined search for computable solutions to ordinary differential equations. The authors humorously called the method, they have developed, a method "by a thousand monkeys." Nevertheless, to solve important scientific or technical problem, we need solve such systems of equations. In this paper, we would like to illustrate an approach that sometimes can help to find solution(s). At its heart are the details of the proof of the theorem on necessary and sufficient conditions for computable solution existence to equations of this type ([5], [6]). The proposed method consists of the selection of replacements of occurrences of the unknown functions in expressions with properly matched functionals and then applying the theorem on the joint fixed point. For simplicity we will restrict ourselves with a simple case that shows the needed

steps.
Example. Suppose we are given the following transcendental equation:

$$[f(n)/2] + (f(n))^{2f(n-1)} \simeq (2f(n-2))^{f(n+1)} + f(n-2). \tag{3}$$

Easy to see that the main obstacles are the presence of the "Integer part" and summation functions. To eliminate these parts, We define an auxiliary functional H as follows:

$$H[g](n)) \simeq 2g(n-2). \tag{4}$$

Replacing in (3) the occurrence of $f(n)$ with the definition of $H[f](n)$ and making cancellation, we get:

$$(2f(n-2)^{2f(n-1)} \simeq (2f(n-2))^{f(n+1)}. \tag{5}$$

At the next step we replace in (5) $f(n+1)$ with $H[f](n+1)$ and obtain

$$(2f(n-2)^{2f(n-1)} \simeq (2f(n-2))^{2f(n-1)}. \tag{6}$$

that is it is an identity. Easy to see that the S.C. Kleene's [3] fixed points of the equation (4) are the functions satisfying the condition

$$f(n) \simeq 2f(n-2). \tag{7}$$

Consequently, as a solution to (3) we can take any function satisfying the definition (7) (adding initial values).

References

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