

Combined Digital Methods for Solving Optimal Resource Allocation Problems

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Abstract

This article is devoted to the development of new effective methods for solving optimal resource allocation problems. The simulated annealing and genetic methods of digital optimization are widely used for solving these kinds of problems. Though these methods approach to the optimal solution of the problem, as usual, a long period of time is required for obtaining the exact solution. This article offers to combine the simulated annealing method with the modification of downhill simplex method to increase the convergence of the optimization method.

Keywords: Simulated annealing, Downhill simplex, Modification, Allocation, Resources.

1. Introduction

The optimal usage of industrial, technological, financial and other resources is one of the important preconditions for further development of the society. The equal allocation of resources during the period of their usage is a special kind of optimal resource distribution. These kinds of problems arise during the parallel implementation of many industrial and technological processes as well as in the fields of construction engineering, financial investments, labor force distribution and so on. In such fields it is often required to work with stable number of employees, provide equal funding in a certain period of time, etc. Usually these are the problems of digital optimization.

According to the analysis of the recent scientific articles, researchers give preference to the meta-heuristic methods, particularly to the genetic, simulated annealing and other methods to solve these kinds of problems [1, 2]. These methods approach to the surroundings of global minimum, however, they require a great number of iterations for obtaining the exact global minimum of the objective function [3, 4]. That is why many researchers are satisfied with finding an approximate solution of optimization problems. So, it is necessary to develop new high performed and efficient methods and algorithms to find optimal solutions to the problems of equal distribution of resources. In this article the modification of downhill simplex method is combined with the simulated annealing method to speed up the search process for finding the optimal solution to these problems.

2. Simulated Annealing Method

The simulated annealing method is a meta-heuristic method used for finding the minimal value of the given objective function $f(\bar{x})$, where $\bar{x} \in R^n$. The method allows to approach the global minimum of the given function in conditions of presence of several local minima. This method is based on crystallization processes of slow freezing thermodynamic systems, such as metals [5]. During the gradual decrease of temperature, the thermodynamic system shifts from the higher energy state to the lower energy state and comes to more stable structure. Sometimes the system shifts towards the greater energy state, the result of which is more unstable metal structure. Then the system changes its energy state again and comes to more stable condition.

By the analogy of thermodynamic systems, the simulated annealing method does not permanently move towards the reduction of objective function values during the minimization process. The method sometimes allows moving to the direction of increment of the function values, depending on some probability. To determine the optimal value of objective function $f(\bar{x})$ the following steps are done by this method. At first point $\bar{x}_{cur} \in R^n$ is randomly selected from the search space of function $f(\bar{x})$ and then initial temperature T is determined. Then the temperature T is gradually decreased while it is higher than the already given temperature $T_{min} > 0$.

For each value of temperature T the following cycle is performed.

The new point $\bar{x}_{new} \in R^n$ is chosen from the search space of the objective function. Then $\Delta f = f(\bar{x}_{new}) - f(\bar{x}_{cur})$ difference is determined.

1. When $\Delta f < 0$, in sense of minimization, the new value of function $f(\bar{x})$ is "better" than the current value. In this case the new point \bar{x}_{new} is definitely accepted as the current point of the search space of the objective function.

2. When $\Delta f > 0$, the objective function worsens its value on the new point. In this case the new point \bar{x}_{new} is accepted by probability $p = e^{-\Delta f/kT}$, despite of the fact that the function takes higher value on that point (k is the constant of Boltzmann). This step allows to observe wider areas.

In the end the point is displayed on which the objective function takes minimal value.

Simulated Annealing Algorithm (Algorithm 1)

A. Initialization

Step 1.1. Generate a random point $\bar{x}_{cur} \in R^n$ from the search space of the objective function $f(\bar{x})$, $\bar{x} \in R^n$. Choose \bar{x}_{opt} as the best point. Assign the point \bar{x}_{cur} to the point \bar{x}_{opt} , $\bar{x}_{opt} := \bar{x}_{cur}$.

Step 1.2. Select the minimal temperature $T_{min} > 0$, the initial temperature $T > 0$ and the value of parameter $h \in (0; 1)$. In [3] the following values are given: $T_{min} = 10^{-4}$, $T = \left| \frac{f(\bar{x}_{cur})}{\ln(0.9)} \right|$, $h = 0.95$.

B. Main Cycle

While $T > T_{min}$, repeat steps 2.1-2.5.

Step 2.1. Generate the new random point $\bar{x}_{new} \in R^n$ from the search space of the function $f(\bar{x})$.

Step 2.2. Determine the difference of values of the function $f(\bar{x})$ on points \bar{x}_{new} and \bar{x}_{cur} .

$$\Delta f := f(\bar{x}_{new}) - f(\bar{x}_{cur})$$

Step 2.3. If $\Delta f \leq 0$, accept the point \bar{x}_{new} as a current point of the search space, $\bar{x}_{cur} := \bar{x}_{new}$.

Moreover, if $f(\bar{x}_{new}) < f(\bar{x}_{opt})$, accept the point \bar{x}_{new} as the optimum point of that moment. $\bar{x}_{opt} := \bar{x}_{new}$. Go to step 2.5.

Step 2.4. If $\Delta f > 0$, decide whether to accept the new worse point as a current point of the search space or not depending on some probability. Do steps 2.4.1-2.4.3.

2.4.1. Generate the random number $r \in [0; 1]$.

2.4.2. Calculate $p = e^{-\frac{\Delta f}{kT}}$ probability, where $k = 1.380650524 * 10^{-23}$.

2.4.3. If $r \leq p$, accept \bar{x}_{new} as a current point, $\bar{x}_{cur} := \bar{x}_{new}$.

Step 2.5. $T := h \cdot T$ (Decrease the current temperature of the system).

C. Final Stage

Step 3.1. Display the vector \bar{x}_{opt} and the value $f(\bar{x}_{opt})$ as the best value of the objective function. Terminate.

In this case there is a high probability that the point \bar{x}_{opt} is quite close to the global minimum of the objective function. It is necessary to take higher initial temperature T , and lower T_{min} in order to get exact results. This step significantly increases the number of calculations. To overcome this disadvantage, the simulated annealing method is combined with downhill simplex method in [3]. In this article the simulated annealing method is combined with the modification of downhill simplex method in order to increase the convergence of the proposed method.

3. The Modification of Downhill Simplex Method

The downhill simplex method is a method of local optimization for finding minimal value of non-linear objective function $f(\bar{x})$, where $\bar{x} \in R^n$. The method is initially proposed by Nelder and Mead [6]. Simplex is a convex set with $n+1$ vertices in n -dimensional space. At first, the regular simplex $S = \{\bar{x}_i\}$ is constructed in the search space of the objective function, $\bar{x}_i \in R^n$, $i = \overline{1, n+1}$. Then, the center \bar{x}_c of simplex S is determined, by formula $\bar{x}_c = \frac{1}{n} \sum_{i=1}^n \bar{x}_i$. The center \bar{x}_c is disposed equally from the first n vertices in the initial method. The \bar{x}_{n+1} vertex of simplex is replaced with the point which decreases the value of the objective function and belongs to the line connecting \bar{x}_c and \bar{x}_{n+1} .

The suggested modification is based on the idea of weighting coefficients. In this case, the search process of the minimum value of the objective function occurs towards the direction in which the values of the given function decrease at high speed. In this case the weighted center \bar{x}'_c of simplex is determined by formula $\bar{x}'_c = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 + \dots + \lambda_n \bar{x}_n$, where $\sum_{i=1}^n \lambda_i = 1$, $\lambda_i > 0$, $i = \overline{1, n}$.

It's proposed to choose such values for coefficients λ_i which will make the weighted center \bar{x}'_c tilt to the vertex of simplex, towards which the function values decrease at high speed. Then, the vertex \bar{x}_{n+1} is replaced with the point which decreases the value of objective function and belongs to the line connecting the points \bar{x}'_c and \bar{x}_{n+1} . The detailed description of the suggested modification of the downhill simplex method is given in the publication [7].

Modified Downhill Simplex Algorithm (Algorithm 2)

A. Initialization

Step 1.1. Generate the random point $\bar{x}_1 \in R^n$ from the search space of the objective function $f(\bar{x})$.

Step 1.2. Around the point \bar{x}_1 , construct the regular simplex $S=\{\bar{x}_i\}, i = \overline{1, n+1}$ with the given rib $h>0$. Choose the sufficiently small number $\varepsilon > 0$, for example $\varepsilon = 10^{-8}$.

B. Main Cycle

While $|f(\bar{x}_{n+1}) - f(\bar{x}_1)| > \varepsilon$ or $\rho(\bar{x}_1, \bar{x}_{n+1}) = \sqrt{\sum_{j=1}^n (x_{n+1,j} - x_{1,j})^2} > \varepsilon$, the steps 2-7 are done, otherwise the step 8 is done.

Step 2. The vertices of the simplex are sorted to meet condition $f(\bar{x}_1) \leq f(\bar{x}_2) \leq \dots \leq f(\bar{x}_{n+1})$.

Step 3. Determine the coefficients $\mu_i = (f(\bar{x}_{n+1}) - f(\bar{x}_i)) / \rho(\bar{x}_{n+1}, \bar{x}_i)$, where $\rho(\bar{x}_{n+1}, \bar{x}_i) = \sqrt{\sum_{j=1}^n (x_{n+1,j} - x_{i,j})^2}$ is the distance between vertices \bar{x}_{n+1} and $\bar{x}_i = (x_{i,1}, x_{i,2}, \dots, x_{i,n}) \in R^n$, where μ_i shows the relative change of function $f(\bar{x})$ on the vertices \bar{x}_{n+1} and $\bar{x}_i, i = \overline{1, n}$.

Step 4. Determine new coefficients λ_i by normalizing weighted coefficients $\mu_i, i = \overline{1, n}$. Choose $\lambda_i = \mu_i / \mu, i = \overline{1, n}$, where $\mu = \sum_{i=1}^n \mu_i$. Coefficients λ_i meet the condition $\sum_{i=1}^n \lambda_i = 1$.

Step 5. Determine the weighted center of simplex by formula $\bar{x}'_c = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 + \dots + \lambda_n \bar{x}_n$.

Step 6. A new point, which decreases the value of the objective function $f(\bar{x})$, is being searched on the line $l_2 = \{\bar{x}'_c + \alpha (\bar{x}'_c - \bar{x}_{n+1})\}$.

Step 6.1. For this purpose, on the line l_2 the points $\bar{x}_r, \bar{x}_e, \bar{x}_{oc}, \bar{x}_{ic}$ are observed, corresponding to the values $\{1; 2; 0.5; -0.5\}$ of α .

Step 6.2. For the mentioned points the following condition is checked by order: if on the new point the value of the function $f(\bar{x})$ is lower, than on \bar{x}_{n+1} point, replace vertex \bar{x}_{n+1} of the simplex with that point, go back to the step 2. If there is no replacement move to the step 7.

Step 7. Shrink the simplex to the vertex \bar{x}_1 by formula $\bar{x}_i = \bar{x}_1 + \sigma(\bar{x}_i - \bar{x}_1), i=2, \dots, n+1, \sigma = 0.5$.

C. Final Stage

Step 8. Display the vector \bar{x}_1 and the optimal value $f(\bar{x}_1)$. End the algorithm.

4. The Proposed Combined Algorithm (algorithm 3)

In order to reach the neighborhood of the objective function $f(\bar{x}), \bar{x} \in R^n$, a certain number of steps of simulated annealing algorithm are done, by choosing $T_{min} = 10^{-2}$. In the result the vector $\bar{x}_{opt} \in R^n$ is obtained, which is quite close to the global minimum of the objective function. Then, a regular simplex is constructed around the point $\bar{x}_{opt} \in R^n$ and the proposed modification of downhill simplex method is applied.

A. Description of the Proposed Combined Algorithm

The following steps are done to get the minimum value of the objective function $f(\bar{x}), \bar{x} \in R^n$.

1. Perform a certain number of steps of simulated annealing algorithm (algorithm 1), by choosing $T_{min} = 10^{-2}$. Obtain $\bar{\mathbf{x}}_{opt} \in R^n$ vector in the result of this algorithm.
2. Assign $\bar{\mathbf{x}}_1 := \bar{\mathbf{x}}_{opt}$. By this step the vector $\bar{\mathbf{x}}_{opt}$, obtained by the simulated annealing method, is chosen as the first $\bar{\mathbf{x}}_1 \in R^n$ vertex of the constructed initial simplex.
3. Apply the modified downhill simplex algorithm (algorithm 2), starting from the step 1.2.
4. In the end of the modified downhill simplex method, display the obtained vector $\bar{\mathbf{x}}_1$ and the value $f(\bar{\mathbf{x}}_1)$ as the minimal value of the objective function.

This combined method has been tested on standard test functions of global optimization listed in [3]. A hundred independent tests have been done to solve each task and attempt the optimal value. Every time the algorithm was started from different points of the search space of test function.

The proposed combined method was compared with the combination of the simulated annealing method and Nelder-Mead's downhill simplex method. Table 1 shows the comparative evaluations between the two combined methods where the **success rate** shows the number of the successful cases of determining the global minimum of each standard function during the 100 independent tests. The test is considered successful and the minimal value of the test functions is achieved if condition $|f^* - \hat{f}| < 10^{-4} \cdot |f^*| + 10^{-6}$ is satisfied [3]. Here f^* is the exact global minimum of the test function and \hat{f} is the optimal value obtained by the combined methods.

Table 1. Combination of the simulated annealing method with the initial and modified downhill simplex methods.

| N | Test function | n (number of variables) | Combination with the initial downhill simplex method | Combination with the modified downhill simplex method |
|----|-----------------------|----------------------------------|--|---|
| | | | Success rate | Success rate |
| 1 | Rosenbrock's valley | 4 | 80% | 85% |
| 2 | Freudenstein and Roth | 2 | 82% | 87% |
| 3 | Shubert | 2 | 86% | 89% |
| 4 | Griewank | 2 | 100% | 100% |
| 5 | Griewank | 4 | 100% | 100% |
| 6 | Griewank | 6 | 100% | 100% |
| 7 | Goldstein and Price | 2 | 100% | 100% |
| 8 | Bohachevsky 2 | 2 | 82% | 85% |
| 9 | Bohachevsky 3 | 2 | 88% | 91% |
| 10 | Schwefel | 2 | 78% | 82% |
| 11 | Six-hump camel back | 2 | 100% | 100% |

Table 1 shows that the combination of the simulated annealing method with the modified downhill simplex method has better results than the combination with Nelder-Mead initial method. During the hundred independent tests the proposed modified combined method in the algorithm 3 has obtained the global minimum of the test function more often than the initial combined method. For example, the number of the cases of obtaining the global minimum of Rosenbrock's valley, Freudenstein and Roth test functions is increased by 5%, while for Shubert, Bohachevsky, Schwefel functions it is increased by 3-4%.

5. The Application of the Proposed Combined Method for Solving Problems of Equal Allocation of Resources in Time

The proposed combined method is applicable for the solution of the following problem of the optimal resource distribution.

Problem 1: n number of industrial, technological or other type of projects are accomplished. The given $f_i(t)$, $t \in [0, d_i]$, $i = \overline{1, n}$ functions show the number of the required resources for execution of the project i at time t . For each project the duration d_i is given. It is allowed to postpone the implementation of the given projects. c_i is the earliest and v_i is the latest start allowed for the project i . If the project i starts with the delay $t_i \in [c_i, v_i]$ the function describing the allocation of resources at time t is $f_i(t - t_i)$ where $t \in [t_i, t_i + d_i]$, $i = \overline{1, n}$. In this case, the sum function $\sum_{i=1}^n f_i(t - t_i)$ will present the total number of resources required at time t . It is required to select such values of t_1, t_2, \dots, t_n start points which allow the equal allocation of the total resources during the given time.

A. The Selection of Criteria for Equal Allocation of Resources

We take the following criteria for the equal allocation of resources: $I_1(t_1, t_2, \dots, t_n)$ and $I_2(t_1, t_2, \dots, t_n)$, where $I_1(t_1, t_2, \dots, t_n) = \max_{a \leq t \leq b} |\sum_{i=1}^n f_i(t - t_i) - M|$ and $I_2(t_1, t_2, \dots, t_n) = \sqrt{\frac{1}{b-a} \cdot \int_a^b [\sum_{i=1}^n f_i(t - t_i) - M]^2 dt}$. These criteria represent the maximal deviation of sum function $\sum_{i=1}^n f_i(t - t_i)$ from its mean value M and mean square deviation, correspondingly. The mean value M of required resources during the implementation period of projects is determined by $M = \frac{1}{b-a} \cdot \int_a^b \sum_{i=1}^n f_i(t - t_i) dt$, $a = \min_i t_i = 0$, $b = \max_i t_i + d_i$, where $(b-a)$ is the implementation period of the projects. Equal allocation of sum resources will be obtained in such values of parameters t_1, t_2, \dots, t_n for which criteria $I_1(t_1, t_2, \dots, t_n)$ or $I_2(t_1, t_2, \dots, t_n)$ will accept their minimal value. So the given problem comes to the following problems ($k=1, 2$).

$$\min_{t_1, t_2, \dots, t_n} I_k(t_1, t_2, \dots, t_n),$$

for $c_i \leq t_i \leq v_i$, $i = \overline{1, n}$ restrictions.

The general formulation of the given problem is applicable for the following particular problems.

Problem 1: The system of n parallel operating aggregates is given. Given functions $f_i(t)$, $t \in [0, d_i]$, $i = \overline{1, n}$ show the workload of aggregate i at time t . It is required to select such start points $t_i \in [c_i, v_i]$ for operating the given aggregates which will make the total workload of the system more equal during the time.

Problem 2: n number of parallel implenting projects are given. Given functions $f_i(t)$, $t \in [0, d_i]$, $i = \overline{1, n}$ show the labor force required for the work i at time t . It is required to select such start points $t_i \in [c_i, v_i]$ for the implementation of the projects which will make the distribution of the total labor force more equal during the time.

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Համակցված թվային մեթոդներ՝ ռեսուրսների օպտիմալ բաշխման խնդիրների լուծման համար

Հ. Դերձյան

Ամփոփում

Հոդվածը նվիրված է ըստ ժամանակի ռեսուրսների օպտիմալ բաշխման խնդիրների լուծման արդյունավետ մեթոդների մշակմանը: Նշված խնդիրների լուծման տարածված մեթոդներից են թրծման մոդելավորման, գենետիկական մեթոդները: Այս մեթոդները սովորաբար մոտենում են խնդրի օպտիմալ լուծմանը, բայց ճշգրիտ լուծում գտնելու համար նրանցից պահանջվում է բավական երկար ժամանակ: Մույն հոդվածի մեջ առաջակվում է թրծման մոդելավորման մեթոդը համակցել սիմպլեքս պլանավորման մեթոդի վերափոխման հետ՝ նշված խնդիրների լուծումը ճշգրտելու նպատակով:

Комбинированные численные методы для решения проблем оптимального распределения ресурсов

А. Дерцян

Аннотация

Статья посвящена разработке эффективных методов для решения проблем оптимального распределения ресурсов по времени. Для решения этих проблем широко используются метод отжига, генетический метод и другие. Эти методы приближаются к решению данных задач, но для нахождения точного решения требуется много времени. В этой статье предлагается скомбинировать метод отжига с модифицированным методом последовательного симплексного планирования, с целью нахождения более точного решения данных задач.