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Հայաստանի Հանրապետության Գիտությունների ազգային ակադեմիայի
Ինֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտ

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Կոմպյուտերային գիտության մաթեմատիկական խնդիրներ

Математические проблемы компьютерных наук

Mathematical Problems of Computer Science

LV

ՀՐԱՏԱՐԱԿՎԱԾ Է ՀՀ ԳԱԱ ԻՆՖՈՐՄԱՏԻԿԱՅԻ ԵՎ ԱՎՏՈՄԱՏԱՑՄԱՆ ՊՐՈԲԼԵՄՆԵՐԻ ԻՆՍՏԻՏՈՒՏԻ ԿՈՂՄԻՑ
ОПУБЛИКОВАНО ИНСТИТУТОМ ПРОБЛЕМ ИНФОРМАТИКИ И АВТОМАТИЗАЦИИ НАН РА
PUBLISHED BY INSTITUTE FOR INFORMATICS AND AUTOMATION PROBLEMS OF NAS RA

ԵՐԵՎԱՆ **2021** **YEREVAN**

Կոմպյուտերային գիտության մաթեմատիկական խնդիրներ պարբերականը հրատարակվում է տարեկան երկու անգամ ՀՀ ԳԱԱ Ինֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտի (ԻԱՊԻ) կողմից: Այն ընդգրկում է տեսական և կիրառական մաթեմատիկայի, ինֆորմատիկայի և հաշվողական տեխնիկայի ժամանակակից ուղղությունները:

Այն ընդգրկված է Բարձրագույն որակավորման հանձնաժողովի ընդունելի ամսագրերի ցանկում:

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Он включен в список допустимых журналов Высшей квалификационной комиссии.

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Պարբերականի սույն պրակը նվիրվում է ֆիզիկա-մաթեմատիկական գիտությունների դոկտոր, պրոֆեսոր Եվգենի Արմենակի Հարությունյանին: Այս տարվա մայիսին՝ նրա 85-ամյակին, ցավոք, հաջորդեց նրա մահվան լուրը:

Этот выпуск журнала посвящен доктору физико-математических наук, профессору Евгению Арменаковичу Арутюнян. В мае этого года, его 85-летию, к сожалению, последовало известие о его смерти.

The current volume is dedicated to Professor Evgueni A. Haroutunian, Doctor of Physical and Mathematical Sciences. Unfortunately, in May 2021, the news of his death followed his 85th anniversary.

Professor Evgueni A. Haroutunian, a recognized scientist and respected teacher, Chief Researcher of the Institute for Informatics and Automation Problems of the NAS of RA, Founder of the Scientific School of Information Theory in Armenia, was the Deputy Editor of Transactions of IIAP NAS RA "Mathematical Problems of Computer Science" for many years.

Professor Haroutunian achieved great scientific achievements during his long and fruitful activity. His contribution to the development of Information Theory and Mathematical Statistics is valuable. His scientific results have been repeatedly cited by well-known scientists, included in basic monographs on information theory and in university textbooks.

He had been working in IIAP of NAS of RA since 1958 and leading the department for Information Theory and Applied Statistics for 25 years. Professor Haroutunian conducted pedagogical activities in the leading universities of Armenia, making a significant contribution to the training of young high-quality staff.

For his great service to country and science E. Haroutunian was awarded a gold medal of National Academy of Sciences of Armenia, the Medal of Honour of the Republic of Armenia.

Evgueni Haroutunian achieved the Life Member status, as a sign of his long-term loyal membership and support to IEEE. He had also been a member of the International Statistical Institute, the Institute of Mathematical Statistics and the Bernoulli Society for Mathematical Statistics and Probability for many years.

The Professor always enjoyed the love and respect, as well as the great reputation of the staff and students and will remain in their thoughts and memories for long.

CONTENTS

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A Theorem on Even Pancyclic Bipartite Digraphs

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Abstract

We prove a Meyniel-type condition and a Bang-Jensen, Gutin and Li-type condition for a strongly connected balanced bipartite digraph to be even pancyclic.

Let D be a balanced bipartite digraph of order $2a \geq 6$. We prove that

(i) If $d(x) + d(y) \geq 3a$ for every pair of vertices x, y from the same partite set, then D contains cycles of all even lengths $2, 4, \dots, 2a$, in particular, D is Hamiltonian.

(ii) If D is other than a directed cycle of length $2a$ and $d(x) + d(y) \geq 3a$ for every pair of vertices x, y with a common out-neighbor or in-neighbor, then either D contains cycles of all even lengths $2, 4, \dots, 2a$ or $d(u) + d(v) \geq 3a$ for every pair of vertices u, v from the same partite set. Moreover, by (i), D contains cycles of all even lengths $2, 4, \dots, 2a$, in particular, D is Hamiltonian.

Keywords: Digraphs, Hamiltonian cycles, Bipartite digraphs, Pancyclic, Even pancyclic.

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1. Introduction

In this paper, we consider finite digraphs without loops and multiple arcs. We assume that the reader is familiar with the standard terminology on digraphs and refer the reader to [1]. Every cycle and path is assumed simple and directed. A cycle in a digraph D is called *Hamiltonian* if it includes all the vertices of D. A digraph D is *Hamiltonian* if it contains a Hamiltonian cycle. A digraph D of order $n \geq 3$ is *pancyclic* if it contains cycles of every length k, $3 \leq k \leq n$.

There are numerous sufficient conditions for the existence of a Hamiltonian cycle in a digraph (see, e.g., [1] - [10]). It was proved (see, e.g., [1], [6], [8], [9], [11] - [14]) that a number of sufficient conditions for a digraph (undirected graph) to be Hamiltonian are also sufficient for the digraph to be pancyclic (with some exceptions). For hamiltonicity, the more general and classical one is the following theorem due to M. Meyniel.

Theorem 1: (Meyniel [10]). *Let D be a strong digraph of order $n \geq 2$. If $d(x)+d(y) \geq 2n-1$ for all pairs of non-adjacent vertices in D, then D is Hamiltonian.*

Notice that Meyniel's theorem is a generalization of Ghouila-Houri's and Woodall's theorems.

A digraph D is a *bipartite* if there exists a partition X, Y of its vertex set into two partite sets such that every arc of D has its end-vertices in different partite sets. It is called *balanced* if $|X| = |Y|$. Following [1], we will say that a balanced bipartite digraph D of order $2a$ is *even pancyclic* (note that a number of authors use the term *"bipancyclic"* instead of *"even pancyclic"*) if it contains cycles of all even lengths $4, 6, \dots, 2a$.

An analogue of Meyniel's theorem for the hamiltonicity of balanced bipartite digraphs was given by Adamus et al. [3].

Theorem 2: (Adamus et al. [3]). *Let D be a balanced bipartite digraph of order $2a \geq 4$. Then D is Hamiltonian provided one of the following holds:*

(a) $d(x) + d(y) \geq 3a + 1$ for each pair of non-adjacent vertices $x, y \in V(D)$;

(b) D is strong and $d(x) + d(y) \geq 3a$ for each pair of non-adjacent vertices $x, y \in V(D)$;

(c) the minimal degree of D is at least $(3a + 1)/2$;

(d) D is strong, and the minimal degree of D is at least $3a/2$.

Meszka [15] investigated the even pancyclicity of a balanced bipartite digraph satisfying a weaker condition than those in Theorem 2(a). He proved the following theorem.

Theorem 3: (Meszka [15]). *Let D be a balanced bipartite digraph of order $2a \geq 4$. Suppose that $d(x) + d(y) \geq 3a + 1$ for each two distinct vertices x, y from the same partite set. Then D contains cycles of all even lengths $4, 6, \dots, 2a$.*

Let x, y be a pair of distinct vertices in a digraph D. The pair $\{x, y\}$ is a *dominated pair* (respectively, *dominating pair*) if there is a vertex $z \in V(D) \setminus \{x, y\}$ such that $z \to \{x, y\}$ (respectively, $\{x, y\} \to z$). We will say that a pair of vertices $\{u, v\}$ is a *good* pair if it is dominated or dominating. In this case we will say that u (respectively, v) is a partner of v (respectively, u). In [5], Bang-Jensen et al. gave a new type condition for a digraph to be Hamiltonian. In the same paper, they also conjectured the following strengthening of Meyniel's theorem.

Conjecture 1: *Let D be a strong digraph of order n. Suppose that $d(x) + d(y) \geq 2n - 1$ for every good pair of non-adjacent distinct vertices x, y. Then D is Hamiltonian.*

They also conjectured that this can even be generalized to the following:

Conjecture 2:. *Let D be a strong digraph of order n. Suppose that $d(x) + d(y) \geq 2n - 1$ for every pair of non-adjacent distinct vertices x, y with a common in-neighbor. Then D is Hamiltonian.*

In [5] and [4], it was proved that Conjecture 1 (2) is true if we also require an additional condition.

Theorem 4: (Bang-Jensen et al. [5]). *Let D be a strong digraph of order $n \geq 2$. Suppose that $min\{d(x), d(y)\} \geq n - 1$ and $d(x) + d(y) \geq 2n - 1$ for any pair of non-adjacent vertices x, y with a common in-neighbor. Then D is Hamiltonian.*

In [4], it was proved that if in Conjecture 1 we replace the degree condition $d(x)+d(y) \geq 2n-1$ with $d(x)+d(y) \geq 5n/2-4$, then Conjecture 1 is true.

There are some versions of Conjecture 1 and 2 for balanced bipartite digraphs. (see, e.g., Theorems 5, 6 and 7).

Theorem 5: (Adamus [2]). *Let D be a strong balanced bipartite digraph of order $2a \geq 6$. If $d(x)+d(y) \geq 3a$ for every good pair of distinct vertices x, y, then D is Hamiltonian.*

An analogue of Theorem 4 was given by Wang [16], and recently strengthened by the author [17].

Theorem 6: (Wang [16]). *Let D be a strong balanced bipartite digraph of order $2a \geq 4$. Suppose that, for every dominating pair of vertices $\{x, y\}$, either $d(x) \geq 2a-1$ and $d(y) \geq a+1$ or $d(y) \geq 2a-1$ and $d(x) \geq a+1$. Then D is Hamiltonian.*

Before stating the next theorem we need to define a digraph of order eight.

Example 1: *Let $D(8)$ be the bipartite digraph with partite sets $X = \{x_0, x_1, x_2, x_3\}$ and $Y = \{y_0, y_1, y_2, y_3\}$, and $A(D(8))$ contains exactly the arcs y_0x_1, y_1x_0, x_2y_3, x_3y_2 and all the arcs of the following 2-cycles: $x_i \leftrightarrow y_i$, $i \in [0, 3]$, $y_0 \leftrightarrow x_2$, $y_0 \leftrightarrow x_3$, $y_1 \leftrightarrow x_2$ and $y_1 \leftrightarrow x_3$.*

It is not difficult to check that $D(8)$ is strongly connected, $max\{d(x), d(y)\} \geq 2a-1$ for every pair of vertices $\{x, y\}$ with a common out-neighbor, but it is not Hamiltonian.

Indeed, if C is a Hamiltonian cycle in $D(8)$, then C would contain the arcs x_1y_1 and x_0y_0 and therefore, the path $x_1y_1x_0y_0$ or the path $x_0y_0x_1y_1$, which is impossible since $N^-(x_0) = N^-(x_1) = \{y_0, y_1\}$.

Theorem 7: (Darbinyan [17]). *Let D be a strong balanced bipartite digraph of order $2a \geq 8$. Suppose that $max\{d(x), d(y)\} \geq 2a-1$ for every pair of distinct vertices $\{x, y\}$ with a common out-neighbor. Then D is Hamiltonian unless D is isomorphic to the digraph $D(8)$.*

Motivated by the Bondy famous metaconjecture, the author, together with Karapetyan [20],proposed the following problem:

Problem 1: Characterize those digraphs, which satisfy the conditions of Theorem 5 (or 6 or 7) but are not even pancyclic.

This problem for Theorems 6 and 7 was solved by the author [18] (Theorem 8(ii)), and for Theorem 5 by Adamus [19] (Theorem 9).

Theorem 8: *Let D be a strong balanced bipartite digraph of order $2a$.*

(i). (Darbinyan [18]). *If D contains a cycle of length $2a-2$ and $max\{d(x), d(y)\} \geq 2a-2 \geq 6$ for every pair of distinct vertices $\{x, y\}$ with a common out-neighbor, then for every k, $1 \leq k \leq a-1$, D contains a cycle of length $2k$.*

(ii). (Darbinyan [18]). *If D is not a directed cycle of length $2a \geq 8$ and $max\{d(x), d(y)\} \geq 2a-1$ for every pair of distinct vertices $\{x, y\}$ with a common out-*

neighbor, then for every k, $1 \le k \le a$, D contains a cycle of length $2k$ (in particular, D is even pancyclic) unless D is isomorphic to the digraph $D(8)$.

(iii). (Darbinyan and Karapetyan [20]). *Suppose that D is not a directed cycle of length $2a \ge 10$ and $max\{d(x), d(y)\} \ge 2a - 2$ for every pair of distinct vertices $\{x, y\}$ with a common out-neighbor. Then D contains a cycle of length $2a - 2$ unless D is isomorphic to a digraph of order ten, which we specify.*

The following theorem by Adamus (Theorem 9) and the main result of this paper (Theorem 10) were proved simultaneously and independently.

Theorem 9: (Adamus [19]). *Let D be a balanced bipartite Hamiltonian digraph of order $2a \ge 6$ other than a directed cycle of length $2a$. Suppose that $d(x)+d(y) \ge 3a$ for every good pair of distinct vertices x, y. Then D contains cycles of all even lengths $2, 4, \dots, 2a$.*

Theorem 10: *Let D be a strong balanced bipartite digraph of order $2a \ge 6$ with partite sets X and Y. If $d(x) + d(y) \ge 3a$ for every pair of distinct vertices $\{x, y\}$ either both in X or both in Y, then D contains cycles of all even lengths less than or equal to $2a$ (in particular, D is Hamiltonian).*

The last result (Theorem 10) was presented at the "International Conference Dedicated to 90th Anniversary of Sergey Mergelyan", 20-25 May, 2018, Yerevan, Armenia.

Using some arguments of [2] by Adamus, we can prove the following lemma.

Lemma 1: *Let D be a balanced bipartite digraph of order $2a \ge 6$ with partite sets X and Y. Suppose that D is not a directed cycle of length $2a$ and $d(u) + d(v) \ge 3a$ for every good pair of distinct vertices u, v. Then D either is even pancyclic or every pair of distinct vertices $\{x, y\}$ from the same partite set is a good pair.*

The following theorem follows from Theorem 10 and Lemma 1.

Theorem 11: *Let D be a strong balanced bipartite digraph of order $2a \ge 6$ other than a directed cycle of length $2a$. Suppose that $d(x) + d(y) \ge 3a$ for every good pair of distinct vertices x, y. Then D contains cycles of all even lengths $2, 4, \dots, 2a$.*

It is worth to noting that in the proof of Theorem 10 does not use the fact that D is Hamiltonian. Thus, we have a common alternative proof for Theorems 2, 3, 5 and 9. Note that if a balanced bipartite digraph satisfies the condition of Theorem 2(a) (or Theorem 2(c)), then D is strong.

Example 2: *For any even integer $a \ge 2$ there is a non-strongly connected balanced bipartite digraph D of order $2a$ with partite sets X and Y, such that $d(x)+d(y) \ge 3a$ for every pair of distinct vertices $\{x, y\}$ either both in X or both in Y ,i.e., if D is not strong, then Theorem 10 is not true.*

To see this, we take two balanced bipartite complete digraphs both of order a (a is even) with partite sets U, V and Z, W, respectively. By adding all the possible arcs from Z to V and from W to U we obtain a digraph D. It is easy to check that $d(x) + d(y) \ge 3a$ for every pair of non-adjacent distinct vertices $\{x, y\}$ of D, but D is not strongly connected and hence, D is not Hamiltonian.

2. Terminology and Notations

In this paper, we consider finite digraphs without loops and multiple arcs. Terminology and notations not defined here or above are consistent with [1]. The vertex set and the arc set of a digraph D are denoted by $V(D)$ and $A(D)$, respectively. The *order* of D is the number of its vertices. If $xy \in A(D)$, then we also write $x \to y$ and say that x *dominates* y or y is an *out-neighbor* of x and x is an *in-neighbor* of y. If $x \to y$ and $y \to x$ we shall use the notation $x \leftrightarrow y$ ($x \leftrightarrow y$ is called 2-*cycle*). We set $\overrightarrow{a}[x,y] = 1$ if $xy \in A(D)$ and $\overrightarrow{a}[x,y] = 0$ if $xy \notin A(D)$.

If A and B are two disjoint subsets of $V(D)$ such that every vertex of A dominates every vertex of B, then we say that A *dominates* B, denoted by $A \to B$. Similarly, $A \leftrightarrow B$ means that $A \to B$ and $B \to A$. If $x \in V(D)$ and $A = \{x\}$ we sometimes write x instead of $\{x\}$. Let $N^+_D(x)$, $N^-_D(x)$ denote the set of out-neighbors, respectively the set of in-neighbors of a vertex x in a digraph D. If $A \subseteq V(D)$, then $N^+_D(x,A) = A \cap N^+_D(x)$ and $N^-_D(x,A) = A \cap N^-_D(x)$. The *out-degree* of x is $d^+_D(x) = |N^+_D(x)|$ and $d^-_D(x) = |N^-_D(x)|$ is the *in-degree* of x. Similarly, $d^+_D(x,A) = |N^+_D(x,A)|$ and $d^-_D(x,A) = |N^-_D(x,A)|$. The *degree* of the vertex x in D is defined as $d_D(x) = d^+_D(x) + d^-_D(x)$ (similarly, $d_D(x,A) = d^+_D(x,A) + d^-_D(x,A)$). We omit the subscript if the digraph is clear from the context. The subdigraph of D induced by a subset A of $V(D)$ is denoted by $D[A]$.

For integers a and b, $a \le b$, let $[a,b]$ denote the set of all the integers, which are not less than a and are not greater than b.

The path (respectively, the cycle) consisting of the distinct vertices $x_1, x_2, \ldots, x_m$ ($m \ge 2$) and the arcs x_ix_{i+1}, $i \in [1, m-1]$ (respectively, x_ix_{i+1}, $i \in [1, m-1]$, and x_mx_1), is denoted by $x_1x_2 \cdots x_m$ (respectively, $x_1x_2 \cdots x_mx_1$). The *length* of a cycle or a path is the number of its arcs. We say that $x_1x_2 \cdots x_m$ is a path from x_1 to x_m or is an (x_1, x_m)-*path*. If a digraph D contains a path from a vertex x to a vertex y we say that y is *reachable* from x in D. In particular, x is reachable from itself.

We denote by $K^*_{a,b}$ the complete bipartite digraph with partite sets of cardinalities a and b. A digraph D is *strongly connected* (or, just, *strong*) if there exists a path from x to y and a path from y to x for every pair of distinct vertices x, y. Two distinct vertices x and y are *adjacent* if $xy \in A(D)$ or $yx \in A(D)$ (or both).

Let D be a bipartite digraph with partite sets X and Y. A *matching* from X to Y (from Y to X) is an independent set of arcs with origin in X and terminus in Y (origin in Y and terminus in X). (A set of arcs with no common end-vertices is called *independent*). If D is balanced, one says that such a matching is *perfect* if it consists of precisely $|X|$ arcs.

3. Preliminaries

In [21] and [11], the author studied pancyclicity of a digraph with the condition of the Meyniel theorem. Before stating the main result of [11] we need to define a family of digraphs.

Definition 1: *For any integers n and m, $(n+1)/2 < m \le n-1$, let Φ^m_n denote the set of digraphs D, which satisfy the following conditions: (i) $V(D) = \{x_1, x_2, \ldots, x_n\}$; (ii)*

$x_n x_{n-1} \dots x_2 x_1 x_n$ is a Hamiltonian cycle in D; (iii) for each k, $1 \leq k \leq n-m+1$, the vertices x_k and x_{k+m-1} are not adjacent; (iv) $x_j x_i \notin A(D)$ whenever $2 \leq i+1 < j \leq n$ and (v) the sum of degrees for any two distinct non-adjacent vertices is at least $2n-1$.

Theorem 12: (Darbinyan [11]). *Let D be a strong digraph of order $n \geq 3$. Suppose that $d(x)+d(y) \geq 2n-1$ for all pairs of distinct non-adjacent vertices x, y in D. Then either (a) D is pancyclic or (b) n is even and D is isomorphic to one of digraphs $K^*_{n/2,n/2}$, $K^*_{n/2,n/2} \setminus \{e\}$, where e is an arbitrary arc of $K^*_{n/2,n/2}$, or (c) $D \in \Phi^m_n$ (in this case D does not contain only a cycle of length m).*

Later, Theorem 12, was also proved independently by Benhocine [22].

Lemma 2: (Adamus et al. [3]). *Let D be a strong balanced bipartite digraph of order $2a \geq 4$ with partite sets X and Y. If $d(x)+d(y) \geq 3a$ for every pair of distinct vertices x, y from the same partite set, then D contains a perfect matching from Y to X and a perfect matching from X to Y.*

Following [15], we give the following definition.

Definition 2: *Let D be a balanced bipartite digraph of order $2a \geq 4$ with partite sets X and Y. Let $M_{y,x} = \{y_i x_i \in A(D) \,|\, i = 1,2,\dots,a\}$ be a perfect matching from Y to X. We define a digraph $D^*[M_{y,x}]$ with vertex set $\{v_1, v_2, \dots, v_a\}$ as follows: each vertex v_i corresponds to a pair $\{x_i, y_i\}$ of vertices in D and for each pair of distinct vertices v_l, v_j, $v_l v_j \in A(D^*[M_{y,x}])$ if and only if $x_l y_j \in A(D)$.*

Let D be a balanced bipartite digraph with partite sets X and Y. Let $M_{y,x}$ be a perfect matching from Y to X in D and $D^*[M_{y,x}]$ be its corresponding digraph. Further, in this paper, we will denote the vertices of D (respectively, of $D^*[M_{y,x}]$) by letters x, y (respectively, u, v) with subscripts or without them.

The *size* of a perfect matching $M_{y,x} = \{y_i x_i \in A(D) \,|\, i = 1,2,\dots,a\}$ from Y to X in D (denoted by $s(M_{y,x})$) is the number of arcs $y_i x_i$ such that $x_i y_i \notin A(D)$.

Using the arguments of [15] by Meszka, we can formulate the following lemma.

Lemma 3: *Let D be a balanced bipartite digraph of order $2a \geq 6$ with partite sets X and Y. Let $M_{y,x} = \{y_i x_i \in A(D) \,|\, i = 1,2,\dots,a\}$ be a perfect matching from Y to X. Then the following hold:*

(i). $d^+(v_i) = d^+(x_i) - \overrightarrow{a}[x_i, y_i]$ and $d^-(v_i) = d^-(y_i) - \overrightarrow{a}[x_i, y_i]$.

(ii). If $D^*[M_{y,x}]$ contains a cycle of length k, where $k \in [2,a]$, then D contains a cycle of length $2k$.

(iii). Suppose that a is even, and $D^*[M_{y,x}]$ is isomorphic to $K^*_{a/2,a/2}$ with partite sets $\{v_1, v_2, \dots, v_{a/2}\}$ and $\{v_{a/2+1}, v_{a/2+2}, \dots, v_a\}$. If D contains an arc from $\{y_1, y_2, \dots, y_{a/2}\}$ to $\{x_{a/2+1}, x_{a/2+2}, \dots, x_a\}$, say $y_{a/2} x_a \in A(D)$, then D contains a cycle of length $2k$ for all $k = 2,3,\dots,a$.

Proof. The proof of Lemma 3 can be found in [15], but we give it here for completeness. (i). It follows immediately from the definition of $D^*[M_{y,x}]$.

(ii). Indeed, if $v_{i_1} v_{i_2} \dots v_{i_k} v_{i_1}$ is a cycle of length k in $D^*[M_{y,x}]$, then $y_{i_1} x_{i_1} y_{i_2} x_{i_2} y_{i_3} \dots$

$y_{i_k}x_{i_k}y_{i_1}$ is a cycle of length $2k$ in D.

(iii). By (ii), it is clear that D contains cycles of every length $4k$, $k = 1, 2, \dots, a/2$. It remains to show that D also contains cycles of every length $4k+2$, $k = 1, 2, \dots, a/2-1$. Indeed, since $x_iy_j \in A(D)$ and $x_jy_i \in A(D)$ for all $i \in [1, a/2]$, $j \in [a/2+1, a]$ and $y_{a/2}x_a \in A(D)$, from the definition of $D^*[M_{y,x}]$ it follows that $y_1x_1y_{a/2+1}x_{a/2+1}y_2x_2y_{a/2+2}x_{a/2+2}y_3$ $x_3 \dots x_ky_{a/2+k}x_{a/2+k}y_{a/2}x_ay_1$ is a cycle of length $4k+2$ in D. $\square$

Lemma 4: (Adamus [2]). Let D be a balanced bipartite digraph of order $2a \geq 6$ other than a directed cycle of length $2a$. Suppose that $d(x) + d(y) \geq 3a$ for every good pair $\{x, y\}$ of distinct vertices in D. Then $d(u) \geq a$ for all $u \in V(D)$.

Now let us prove Lemma 1. For convenience, we will restate it here.

Lemma 1: *Let D be a balanced bipartite digraph of order $2a \geq 6$ with partite sets X and Y. Suppose that D is not a directed cycle of length $2a$ and $d(u) + d(v) \geq 3a$ for every good pair of distinct vertices u, v. Then D either is even pancyclic or every pair of distinct vertices $\{x, y\}$ from the same partite set is a good pair.*

Proof: Let $X = \{x_1, x_2, \dots, x_a\}$ and $Y = \{y_1, y_2, \dots, y_a\}$. Suppose that $V(D)$ contains a pair of vertices from the same partite set, which is not a good pair. Without loss of generality, assume that $\{x_1, x_2\}$ is not a good pair. Then

$$N^+(x_1) \cap N^+(x_2) = N^-(x_1) \cap N^-(x_2) = \emptyset, d^+(x_1) + d^+(x_2) \leq a \ , d^-(x_1) + d^-(x_2) \leq a.$$

Hence, $d(x_1) + d(x_2) \leq 2a$. This together with $d(x_1) \geq a$ and $d(x_2) \geq a$ (Lemma 4) implies that $d(x_1) = d(x_2) = d^+(x_1) + d^+(x_2) = d^-(x_1) + d^-(x_2) = a$. Now we obtain that $N^+(x_1) \cup N^+(x_2) = N^-(x_1) \cup N^-(x_2) = Y$.

Let $x_i \in X \setminus \{x_1, x_2\}$ be an arbitrary vertex. We claim that $\{x_1, x_i\}$ or $\{x_2, x_i\}$ is a good pair. Assume that this is not the case. Then $(N^+(x_1) \cup N^+(x_2)) \cap N^+(x_i) = \emptyset$, which contradicts the facts that D is strong and $N^+(x_1) \cup N^+(x_2) = Y$. Thus, $\{x_1, x_i\}$ or $\{x_2, x_i\}$ is a good pair for all i, $3 \leq i \leq a$. Therefore, from condition (A) and $d(x_1) = d(x_2) = a$ it follows that $d(x_i) = 2a$ for all i, $3 \leq i \leq a$, i.e., $D[X \cup Y \setminus \{x_1, x_2\}]$ is a complete bipartite digraph with partite sets $X \setminus \{x_1, x_2\}$ and Y.

From $d(x_3) = 2a$ it follows that $d^+(x_3) = d^-(x_3) = a$. Therefore, if D contains a Hamiltonian cycle, then D contains cycles of all even lengths $2, 4, \dots, 2a$.

Now we will show that D contains a Hamiltonian cycle.

Assume first that there is an (x_1, x_2)-path of length two. Let $x_1y_1x_2$ be an (x_1, x_2)-path of length two. Then $y_1x_1 \notin A(D)$ and $x_2y_1 \notin A(D)$ as $\{x_1, x_2\}$ is not a good pair. Now, since $x_2y_1 \notin A(D)$ and $d^+(x_2) \geq 1$, we may assume that $x_2y_2 \in A(D)$. From $d^-(x_1)+d^-(x_2) = a \geq 3$ it follows that $d^-(x_1) \geq 2$ or $d^-(x_2) \geq 2$. Assume that $d^-(x_1, \{y_3, y_4, \dots, y_a\}) \geq 1$. We may assume that $y_3x_1 \in A(D)$. Now using the fact that $D[X \cup Y \setminus \{x_1, x_2\}]$ is a complete bipartite digraph, we see that $y_3x_1y_1x_2y_2x_3y_4x_4 \dots y_ax_ay_3$ is a Hamiltonian cycle in D. Assume now that $d^-(x_1, \{y_3, y_4, \dots, y_a\}) = 0$. Then from $y_1x_1 \notin A(D)$ and $d^-(x_1) = 1$ it follows that $y_2x_1 \in A(D)$. Then $x_2y_2x_1$ is an (x_2, x_1)-path of length two and $d^-(x_2) \geq 2$. Now, we have that $y_2x_2 \notin A(D)$ since $\{x_1, x_2\}$ is not a good pair. Therefore, $d^-(x_2, \{y_3, y_4, \dots, y_a\}) \geq 1$. Now, by repeating the above argument, we conclude that D is Hamiltonian. Similarly, one can show that if there is an (x_2, x_1)-path of length two, then again D is Hamiltonian.

Assume next that there is no path of length two between x_1 and x_2. Then $d^-(x_1, N^+(x_2)) = d^-(x_2, N^+(x_1)) = 0$, and from $N^-(x_1) \cup N^-(x_2) = Y$ it follows that $N^-(x_1) = N^+(x_1)$ and $N^-(x_2) = N^+(x_2)$. This together with $d(x_1) = d(x_2) = a$ implies that $|N^+(x_1)| = |N^+(x_2)| = a/2$, a is even and $a \geq 4$. Without loss of generality, we assume that $x_1 \leftrightarrow \{y_1, y_2\}$ and $x_2 \leftrightarrow \{y_{a-1}, y_a\}$. Now, since $D[X \cup Y \setminus \{x_1, x_2\}]$ is a complete bipartite digraph, it is not difficult to check that $x_3y_1x_1y_2x_4y_3x_5y_4 \ldots x_{a-1}y_{a-2}x_ay_{a-1}x_2$ y_ax_3 is a Hamiltonian cycle in D. Thus, in all possible cases, D is Hamiltonian. Lemma 1 is proved. □

4. Proof of the Main Result

Let D be a strong balanced bipartite digraph of order $2a$. We say that D satisfies condition (A) when $d(x) + d(y) \geq 3a$ for all distinct vertices x, y from the same partite set.

The proof of Theorem 10 will be based on the following three lemmas below.

Lemma 5: *Let D be a strong balanced bipartite digraph of order $2a \geq 6$ with partite sets X and Y. If D satisfies condition (A), then D contains cycles of lengths 2 and 4.*

Proof: From condition (A) immediately follows that D contains a cycle of length 2. We will prove that D also contains a cycle of length 4. By Lemma 2, D contains a perfect matching from Y to X. Let $M_{y,x} = \{y_ix_i \in A(D) \,|\, i = 1, 2, \ldots, a\}$ be a perfect matching from Y to X. If for some integers i, j, $1 \leq i \neq j \leq a$, the arcs x_iy_j, x_jy_i are in D, then $x_iy_jx_jy_ix_i$ is a cycle of length 4. We may, therefore, assume that for every pair of integers i, j, $1 \leq i \neq j \leq a$, $\overrightarrow{a}[x_i, y_j] + \overrightarrow{a}[x_j, y_i] \leq 1$. Therefore, for all $i \in [1, a]$,

$$d^-(y_i) \leq a - d^+(x_i) - 1, \text{ if } \overrightarrow{a}[x_i, y_i] = 0 \text{ and } d^-(y_i) \leq a - d^+(x_i) + 1, \text{ if } \overrightarrow{a}[x_i, y_i] = 1. \quad (1)$$

Assume that there are two distinct integers i, j, $1 \leq i, j \leq a$, such that $\overrightarrow{a}[x_i, y_i] = \overrightarrow{a}[x_j, y_j] = 0$. Then, by (1), $d^-(y_i) + d^+(x_i) \leq a - 1$ and $d^-(y_j) + d^+(x_j) \leq a - 1$. These together with condition (A) and the fact that the semi-degrees of every vertex in D are bounded above by a thus implies that

$$6a \leq d(x_i) + d(x_j) + d(y_i) + d(y_j) = d^-(y_i) + d^+(x_i) + d^-(y_j) + d^+(x_j)$$
$$+d^+(y_i) + d^+(y_j) + d^-(x_i) + d^-(x_j) \leq 6a - 2,$$

which is a contradiction.

Assume now that for some $i \in [1, a]$, $\overrightarrow{a}[x_i, y_i] = 0$ and for all $j \in [1, a] \setminus \{i\}$, $\overrightarrow{a}[x_j, y_j] = 1$. Without loss of generality, we may assume that $i = 1$. By (1), $d^-(y_1) + d^+(x_1) \leq a - 1$ and $d^-(y_2) + d^+(x_2) \leq a + 1$. If for some $k \in [3, a]$, $y_2x_k \in A(D)$ and $y_kx_2 \in A(D)$, then $x_2y_2x_ky_kx_2$ is a cycle of length 4 in D. We may, therefore, assume that $\overrightarrow{a}[y_2, x_k] + \overrightarrow{a}[y_k, x_2] \leq 1$ for all $k \in [3, a]$. This implies that

$$d^-(x_2) + d^+(y_2) = d^-(x_2, \{y_1, y_2\}) + d^+(y_2, \{x_1, x_2\}) + d^-(x_2, Y \setminus \{y_1, y_2\})$$
$$+d^+(y_2, X \setminus \{x_1, x_2\}) \leq 4 + a - 2 = a + 2.$$

Using the above inequalities and condition (A), we obtain

$$6a \leq d(x_1) + d(x_2) + d(y_1) + d(y_2) = d^-(y_1) + d^+(x_1) + d^-(y_2) + d^+(x_2)$$

$$+d^-(x_2)+d^+(y_2)+d^-(x_1)+d^+(y_1)\leq 5a+2,$$

which is a contradiction since $a\geq 3$.

Assume finally that $x_iy_i\in A(D)$ for all $i\in[1,a]$. In this case, by the symmetry between the vertices x_i and y_i, similar to (1), we obtain that $d^-(x_i)+d^+(y_i)\leq a+1$. This together with (1) implies that for any i, j $(1\leq i\neq j\leq a)$,

$$6a\leq d(x_i)+d(x_j)+d(y_i)+d(y_j)\leq 4a+4,$$

a contradiction since $a\geq 3$. Lemma 5 is proved. $\square$

Remark 1: There is a strong balanced bipartite digraph of order 4, which satisfies condition (A), but contains no cycle of length 4. To see this, we consider the following digraph with vertex set $V(D)=\{x_1,x_2,y_1,y_2\}$ and arc set $D(A)=\{x_1y_2,y_2x_2,x_2y_2,x_2y_1,$
$y_1x_1\}$.

Lemma 6: *Let D be a strong balanced bipartite digraph of order $2a\geq 6$ with partite sets X and Y. Let $M_{y,x}=\{y_ix_i\in A(D)\,|\,i=1,2,\ldots,a\}$ be a perfect matching from Y to X in D such that the size $s(M_{y,x})$ of $M_{y,x}$ is maximum among the sizes of all the perfect matching from Y to X in D. If D satisfies condition (A), then the digraph $D^*[M_{y,x}]$ either is strong or D contains cycles of all lengths $2,4,\ldots,2a$.*

Proof: Notice that, by Lemma 5, D contains cycles of lengths 2 and 4. Suppose that the digraph $D^*[M_{y,x}]$ is not strong. Then in $D^*[M_{y,x}]$ there are two distinct vertices, say v_1 and v_j, such that there is no path from v_1 to v_j in $D^*[M_{y,x}]$. Let U be the set of all vertices reachable from v_1 and W be the set of all vertices from which v_j is reachable. Notice that $v_1\in U$, $v_j\in W$ and $U\cap W=\emptyset$.

Case 1. $d^+(v_1)\geq 1$ and $d^-(v_j)\geq 1$.

Then $|U|\geq 2$ and $|W|\geq 2$. Let v_l, v_k be two distinct vertices in U and v_p, v_q be two distinct vertices in W. From condition (A) and the fact that the semi-degrees of every vertex in D are bounded above by a it follows that

$$d^+(x_l)+d^+(x_k)\geq a \quad \text{and} \quad d^-(x_p)+d^-(x_q)\geq a. \tag{2}$$

By Lemma 3(i),

$$d^+(v_l)+d^+(v_k)=d^+(x_l)+d^+(x_k)-\overrightarrow{a}[x_l,y_l]-\overrightarrow{a}[x_k,y_k],$$

and

$$d^-(v_p)+d^-(v_q)=d^-(y_p)+d^-(y_q)-\overrightarrow{a}[x_p,y_p]-\overrightarrow{a}[x_q,y_q]. \tag{3}$$

It follows from them and (2) that $d^+(v_l)+d^+(v_k)\geq a-2$ and $d^-(v_p)+d^-(v_q)\geq a-2$. Without loss of generality we may assume that $d^+(v_l)\geq (d^+(v_l)+d^+(v_k))/2$ and $d^-(v_p)\geq (d^-(v_p)+d^-(v_p))/2$. These imply that $d^+(v_l)\geq (a-2)/2$ and $d^-(v_p)\geq (a-2)/2$, which in turn imply that $|U|\geq a/2$ and $|W|\geq a/2$.

If $d^+(v_l)+d^+(v_k)\geq a-1$ or $d^-(v_p)+d^-(v_q)\geq a-1$, then $|U|\geq (a+1)/2$ or $|W|\geq (a+1)/2$, respectively. Hence $|U|+|W|\geq (2a+1)/2$, which is a contradiction since $|U|+|W|\leq a$. Using (2) and (3), we may therefore assume that

$$d^+(v_l)+d^+(v_k)=d^+(x_l)+d^+(x_k)-2=d^-(v_p)+d^-(v_q)=d^-(y_p)+d^-(y_q)-2=a-2.$$

Then it is easy to see that the arcs x_ly_l, x_ky_k, x_py_p and x_qy_q are in D, $|U| = |W| = a/2$ and $V(D^*[M_{y,x}]) = U \cup W$. In particular, a is even. Without loss of generality, we assume that $U = \{v_1, v_2, \ldots, v_{a/2}\}$ and $W = \{v_{a/2+1}, v_{a/2+2}, \ldots, v_a\}$. Since there is no arc from a vertex in U to a vertex in W, the following holds:

$$A(\{x_1, x_2, \ldots, x_{a/2}\} \to \{y_{a/2+1}, y_{a/2+2}, \ldots, y_a\}) = \emptyset. \tag{4}$$

Therefore, if $i \in [1, a/2]$ and $j \in [a/2+1, a]$, then $d^+(x_i) \leq a/2$ and $d^-(y_j) \leq a/2$. Together with (2) they imply that $d^+(x_i) = d^-(y_j) = a/2$ and

$$x_i \to \{y_1, y_2, \ldots, y_{a/2}\} \quad \text{and} \quad \{x_{a/2+1}, x_{a/2+2}, \ldots, x_a\} \to y_j \tag{5}$$

for all $i \in [1, a/2]$ and $j \in [a/2+1, a]$, respectively. Therefore, by condition (A),

$$3a \leq d(x_i) + d(x_k) \leq a + d^-(x_i) + d^-(x_k),$$

for every pair of $i, k \in [1, a/2]$. This implies that $d^-(x_i) = d^-(x_k) = a$, which means that $\{y_1, y_2, \ldots, y_a\} \to \{x_i, x_k\}$. Similarly, $y_j \to \{x_1, x_2 \ldots, x_a\}$, for all $j \in [a/2+1, a]$. From this and (5) it follows that the induced subdigraphs $D[\{x_1, x_2, \ldots, x_{a/2}, y_1, y_2, \ldots, y_{a/2}\}]$ and $D[\{x_{a/2+1}, x_{a/2+2}, \ldots, x_a, y_{a/2+1}, y_{a/2+2}, \ldots, y_a\}]$ both are balanced bipartite complete digraphs. Therefore, D contains cycles of all lengths $2, 4, \ldots, a$. It remains to show that D also contains cycles of every length $a + 2b$, $b \in [1, a/2]$. Since D is strong and (4), it follows that there is an arc from a vertex in $\{y_1, y_2, \ldots, y_{a/2}\}$ to a vertex in $\{x_{a/2+1}, x_{a/2+2}, \ldots, x_a\}$. Without loss of generality, we may assume that $y_{a/2}x_{a/2+1} \in A(D)$. Then $x_1y_1x_2y_2 \ldots x_{a/2}y_{a/2}x_{a/2+1}\ y_{a/2+1}x_{a/2+2} \ldots x_{a/2+b}y_{a/2+b}x_1$ is a cycle of length $a + 2b$. Thus, D contains cycles of all lengths $2, 4, \ldots, 2a$. This completes the discussion of Case 1.

Case 2. $d^+(v_1) = 0$.

Then $d^+(x_1) = 1$ and $x_1y_1 \in A(D)$, since D is strong. Hence $d(x_1) \leq a + 1$. Together with condition A this implies that $a \leq d(x_1) \leq a + 1$. We distinguish two subcases depending on $d(x_1)$.

Case 2.1. $d(x_1) = a$.

Then $d(x_i) \geq 2a$ for all $i \in [2, a]$ because of condition A. Therefore, the induced subdigraph $D\langle Y \cup X \setminus \{x_1\}\rangle$ is a complete bipartite digraph with partite sets Y and $X \setminus \{x_1\}$. It is clear that D contains cycles of every lengths $2, 4, \ldots, 2a - 2$. Since $d(x_1) = a$, $d^+(x_1) = 1$ and $a \geq 3$, we have that $d^-(x_1) = a - 1 \geq 2$. Without loss of generality we may assume that $y_2x_1 \in A(D)$. Then $y_2x_1y_1x_3y_3 \ldots x_ay_ax_2y_2$ is a cycle of length $2a$.

Case 2.2. $d(x_1) = a + 1$.

Then $\{y_1, y_2, \ldots, y_a\} \to x_1$ because of $d^+(x_1) = 1$, and, by condition (A), $d(x_i) \geq 2a - 1$ for all $i \in [2, a]$. Observe that if for some $i \in [2, a]$, $y_1x_i \in A(D)$, then $M^i_{y,x} := \{y_ix_1, y_1x_i\} \cup \{y_jx_j \,|\, j \in [1, a] \setminus \{1, i\}\}$ is a perfect matching from Y to X in D.

Assume that for some $i \in [2, a]$, $x_iy_1 \notin A(D)$. Then $y_1x_i \in A(D)$ because of $d(x_i) \geq 2a - 1$. Since $x_1y_1 \in A(D)$, $x_iy_1 \notin A(D)$ and $x_1y_i \notin A(D)$, it follows that $s(M^i_{y,x}) > s(M_{y,x})$, which contradicts the choice of $M_{y,x}$. We may therefore assume that $\{x_2, x_3, \ldots, x_a\} \to y_1$. If

$y_1x_i \in A(D)$ and $x_iy_i \in A(D)$, where $i \in [2, a]$, then again we have $s(M^i_{y,x}) > s(M_{y,x})$, since the arcs x_1y_1, x_iy_i are in D and $x_1y_i \notin A(D)$. We may therefore assume that $\overrightarrow{a}[y_1, x_i] + \overrightarrow{a}[x_i, y_i] \le 1$. This together with $d(x_i) \ge 2a - 1$, $i \in [2, a]$, implies that

$$\{y_2, y_3, \ldots, y_a\} \to x_i \to \{y_2, y_3, \ldots, y_a\} \setminus \{y_i\}. \tag{6}$$

Since D is strong and $d^+(x_1, \{y_2, y_3, \ldots, y_a\}) = 0$, it follows that $d^+(y_1, \{x_2, x_3, \ldots, x_a\}) \ge 1$. Without loss of generality, we assume that $y_1x_2 \in A(D)$. Then, since $y_2x_1 \in A(D)$ and (6), $x_1y_1x_2y_3x_3 \ldots x_{k-1}y_kx_ky_2x_1$ is a cycle of length $2k$ for every $k \in [3, a]$. Lemma 6 is proved. □

Lemma 7: *Let D be a strong balanced bipartite digraph of order $2a \ge 6$ with partite sets X and Y. Let $M_{y,x} = \{y_ix_i \in A(D) \,|\, i = 1, 2, \ldots, a\}$ be a perfect matching from Y to X in D such that the size $s(M_{y,x})$ of $M_{y,x}$ is maximum among the sizes of all the perfect matching from Y to X in D. If D satisfies condition (A), then either $d(u) + d(v) \ge 2a - 1$ for every pair of non-adjacent vertices u, v in $D^*[M_{y,x}]$ or D contains cycles of all lengths $2, 4, \ldots, 2a$.*

Proof: Suppose that D is not even pancyclic. Then by Lemma 6, $D^*[M_{y,x}]$ is strong. Let v_i and v_j be two arbitrary distinct vertices in $D^*[M_{y,x}]$. Write

$g(i, j) := d^+(x_i) + d^+(x_j) + d^-(y_i) + d^-(y_j)$ and $f(i, j) := d^-(x_i) + d^-(x_j) + d^+(y_i) + d^+(y_j)$.

By Lemma 3(i), we have

$$d(v_i) + d(v_j) = g(i, j) - 2\overrightarrow{a}[x_i, y_i] - 2\overrightarrow{a}[x_j, y_j]. \tag{7}$$

By condition (A), we have

$$6a \le d(x_i) + d(x_j) + d(y_i) + d(y_j) = f(i, j) + g(i, j).$$

Hence,

$$g(i, j) \ge 2a \quad \text{and} \quad 4a \ge f(i, j) \ge 6a - g(i, j). \tag{8}$$

since the semi-degrees of every vertex of D are bounded above by a. Now we prove the following claim.

Claim 1: *Assume that the vertices v_i and v_j in $D^*[M_{y,x}]$ are not adjacent. Then the following hold:*

(i). If $x_iy_i \in A(D)$ or $x_jy_j \in A(D)$, then $\overrightarrow{a}[y_i, x_j] + \overrightarrow{a}[y_j, x_i] \le 1$.

(ii). If $x_iy_i \notin A(D)$ or $x_jy_j \notin A(D)$, then $d(v_i) + d(v_j) \ge 2a - 1$ in $D^*[M_{y,x}]$.

Proof: Since the vertices v_i and v_j in $D^*[M_{y,x}]$ are not adjacent, it follows that $x_iy_j \notin A(D)$ and $x_jy_i \notin A(D)$.

(i). Suppose, to the contrary, that $x_iy_i \in A(D)$ or $x_jy_j \in A(D)$, but $\overrightarrow{a}[y_i, x_j] + \overrightarrow{a}[y_j, x_i] = 2$. Then $M'_{y,x} := \{y_ix_j, y_jx_i\} \cup \{y_kx_k \,|\, k \in [1, a] \setminus \{i, j\}\}$ is a new perfect matching from Y to X in D. Since $x_jy_i \notin A(D)$, $x_iy_j \notin A(D)$ and $x_iy_i \in A(D)$ or $x_jy_j \in A(D)$, it follows that $s(M'_{y,x}) > s(M_{y,x})$, which contradicts the choice of $M_{y,x}$.

(ii). If $\overrightarrow{a}[x_i, y_i] = \overrightarrow{a}[x_j, y_j] = 0$, then from (7) and $g(i, j) \ge 2a$ it follows that $d(v_i) + d(v_j) \ge 2a$ in $D^*[M_{y,x}]$. We may therefore assume that $x_iy_i \in A(D)$. Then $x_jy_j \notin A(D)$ by the assumption of Claim 1(ii). If $g(i, j) \ge 2a+1$, then, by (7), $d(v_i)+d(v_j) \ge 2a-1$.

Thus, we may assume that $g(i,j) = 2a$. Then $f(i,j) \geq 4a$ by (8). The last inequality implies that the arcs y_ix_j, y_jx_i are in D. Therefore, $M'_{y,x} := \{y_ix_j, y_jx_i\} \cup \{y_kx_k \mid k \in [1,a] \setminus \{i,j\}\}$ is a new perfect matching from Y to X in D. Since $x_iy_i \in A(D)$, $x_iy_j \notin A(D)$ and $x_jy_i \notin A(D)$, it follows that $s(M'_{y,x}) > s(M_{y,x})$, which contradicts the choice of $M_{y,x}$. The claim is proved. $\square$

We now return to the proof of Lemma 7. Suppose that there exist two distinct non-adjacent vertices, say v_1 and v_2, in $D^*[M_{y,x}]$ such that

$$d(v_1) + d(v_2) \leq 2a - 2. \tag{9}$$

This together with (7), $\overrightarrow{a}[x_1, y_1] \leq 1$ and $\overrightarrow{a}[x_2, y_2] \leq 1$ implies that $g(1,2) \leq 2a + 2$. Therefore, $2a \leq g(1,2) \leq 2a + 2$.

Case 1. $\overrightarrow{a}[x_1, y_1] = 0$.

Then from (7), (9) and the fact that $g(1,2) \geq 2a$, it follows that $\overrightarrow{a}[x_2, y_2] = 1$ (i.e., $x_2y_2 \in A(D)$) and $g(1,2) = 2a$. From this and (8) it follows that $f(1,2) \geq 4a$, which in turn implies that $y_1x_2 \in A(D)$ and $y_2x_1 \in A(D)$. The aforementioned contradicts Claim 1(i) since $x_2y_2 \in A(D)$.

Case 2. $\overrightarrow{a}[x_1, y_1] = \overrightarrow{a}[x_2, y_2] = 1$, i.e., $x_1y_1 \in A(D)$ and $x_2y_2 \in A(D)$.

From Claim 1(i) it follows that $y_1x_2 \notin A(D)$ or $y_2x_1 \notin A(D)$. If $2a \leq g(1,2) \leq 2a + 1$, then from (8) it follows that $f(1,2) \geq 4a - 1$, which in turn implies that $y_1x_2 \in A(D)$ and $y_2x_1 \in A(D)$, which is a contradiction. We may therefore assume that $g(1,2) = 2a+2$. This and (8) imply that $f(1,2) \geq 4a-2$. Then, since $y_1x_2 \notin A(D)$ or $y_2x_1 \notin A(D)$, it follows that $y_1x_2 \in A(D)$ or $y_2x_1 \in A(D)$. Without loss of generality, we may assume that $y_1x_2 \notin A(D)$ and $y_2x_1 \in A(D)$. Note that the vertices y_1 and x_2 are not adjacent. Then $f(1,2) = 4a - 2$, which in turn implies that $d^-(x_1) = d^+(y_2) = a$ and $d^-(x_2) = d^+(y_1) = a - 1$. Therefore,

$$y_2 \to \{x_1, x_2, \dots, x_a\};\ \{y_1, y_2, \dots, y_a\} \to x_1;\ y_1 \to \{x_1, x_3, x_4, \dots, x_a\};$$

$$\{y_2, y_3, \dots, y_a\} \to x_2. \tag{10}$$

since $y_1x_2 \notin A(D)$. Using (10), it is easy to see that for all $i \in [3,a]$,

$$M^i_{y,x} := \{y_2x_1, y_ix_2, y_1x_i\} \cup \{y_kx_k \mid k \in [3,a] \setminus \{i\}\}$$

is a perfect matching from Y to X in D. Using the facts that the arcs x_1y_1, x_2y_2 are in D, it is not difficult to see that if for some $i \in [3,a]$, either $x_2y_i \notin A(D)$ or $x_iy_1 \notin A(D)$ or $x_iy_i \in A(D)$, then $s(M^i_{y,x}) > s(M_{y,x})$, which contradicts the choice of $M_{y,x}$. We may therefore assume that $x_iy_i \notin A(D)$ for all $i \in [3,a]$, and $x_2 \to \{y_2, y_3, \dots, y_a\}$ and $\{x_3, x_4, \dots, x_a\} \to y_1$. Together with (10) they imply that

$$x_2 \leftrightarrow \{y_2, y_3, \dots, y_a\} \quad \text{and} \quad y_1 \leftrightarrow \{x_1, x_3, x_4, \dots, x_a\}. \tag{11}$$

Since the vertices y_1, x_2 are not adjacent, from (11) and Lemma 3(i) it follows that

$$d^-(y_1) = d^+(x_2) = a - 1, \quad d^-(v_1) = d^+(v_2) = a - 2. \tag{12}$$

From $g(1,2) = 2a + 2$, (7), $x_1y_1 \in A(D)$ and $x_2y_2 \in A(D)$ it follows that $d(v_1) + d(v_2) = g(1,2) - 4 = 2a - 2$. This together with (12) and the fact that $D^*[M_{y,x}]$ is strong implies that

$d^+(v_1) = d^-(v_2) = 1$. This means that $d^+(x_1) = d^-(y_2) = 2$. Therefore, $d(x_1) = d(y_2) = a+2$ by (10).

Now for every $i \in [3, a]$ we consider the perfect matching $M^i_{y,x}$ and its corresponding digraph $D^*[M^i_{y,x}]$. Notice that $s(M_{y,x}) = s(M^i_{y,x}) = a-2$, the vertices y_1, x_2 are not adjacent and the arcs x_iy_i, x_1y_2 are not in $A(D)$. Hence, the vertices $v^i_1 = \{y_1, x_i\}$, $v^i_2 = \{y_i, x_2\}$ in $D^*[M^i_{y,x}]$ are not adjacent. From Claim 1(ii) it follows that in $D^*[M^i_{y,x}]$ the degree sum of every pair of two distinct non-adjacent vertices, other than $\{v^i_1, v^i_2\}$, is at least $2a-1$. If in $D^*[M^i_{y,x}]$, $d(v^i_1) + d(v^i_2) \leq 2a-2$, then by the arguments to that in the proof of $d(x_1) = d(y_2) = a+2$, we deduce that $d(x_i) = d(y_i) = a+2$ for all $i \in [3, a]$. Therefore, for all $i \in [3, a]$, $3a \leq d(x_1) + d(x_i) \leq 2a+4$. This means that $a \leq 4$, i.e., $a = 3$ or $a = 4$.

Let $a = 3$. By Lemma 5, it suffices to show that D contains a cycle of length 6. Using (10) and (11), it is easy to check that $x_3y_2x_2y_3x_1y_1x_3$ is a cycle of length 6 in D.

Let now $a = 4$. By Lemma 5, we need to show that D contains cycles of lengths 6 and 8. From $d(x_4) = 6$ and $x_4y_4 \notin A(D)$ it follows that $x_4y_2 \in A(D)$ or $x_4y_3 \in A(D)$.

Assume that $x_3y_4 \in A(D)$. Then using (10) and (11) it is not difficult to see that $x_3y_4x_2y_2x_1y_1x_3$ is a cycle of length 6, and $x_3y_4x_4y_2x_2y_3x_1y_1x_3$ (respectively, $x_3y_4x_4y_3x_2y_2$ $x_1y_1x_3$) is a cycle of length 8, when $x_4y_2 \in A(D)$ (respectively, when $x_4y_3 \in A(D)$).

Assume now that $x_3y_4 \notin A(D)$. Then from $x_4y_4 \notin A(D)$ and $d(y_4) = 6$ it follows that $x_1y_4 \in A(D)$. Now again using (10) and (11), we see that $x_1y_4x_2y_2x_3y_1x_1$ is a cycle of length 6, and $x_1y_4x_4y_2x_2y_3x_3y_1x_1$ (respectively, $x_1y_4x_4y_3x_2y_2x_3y_1x_1$) is a cycle length 8, when $x_4y_2 \in A(D)$ (respectively, when $x_4y_3 \in A(D)$). Thus, we have shown that if $a = 3$ or $a = 4$, then D contains cycles of all lengths $2, 4, \ldots, 2a$, which contradicts our supposition that D is not even pancyclic. This completes the proof of Lemma 7. $\square$

We now ready to complete the proof of Theorem 10.

Proof of Theorem 10: Let D be a digraph satisfying the conditions of Theorem 10. By Lemma 5, D contains cycles of lengths 2 and 4. By Lemma 2, D contains a perfect matching from Y to X. Let $M_{y,x} = \{y_ix_i \in A(D) \mid i = 1, 2, \ldots, a\}$ be a perfect matching from Y to X in D with the maximum size among the sizes of all the perfect matching from Y to X in D. By Lemma 6, the digraph $D^*[M_{y,x}]$ either contains cycles of all lengths $2, 4, \ldots, 2a$ or is strongly connected. In the former case we are done. Assume that $D^*[M_{y,x}]$ is strongly connected. By Lemma 7, D either contains cycles of all lengths $2, 4, \ldots, 2a$ or (ii) $d(u) + d(v) \geq 2a-1$ for every pair of non-adjacent vertices u, v in $D^*[M_{y,x}]$. Assume that the second case holds. Therefore, by Theorem 12, either (a) $D^*[M_{y,x}]$ contains cycles of every length k, $k \in [3, a]$ or (b) a is even and $D^*[M_{y,x}]$ is isomorphic to one of digraphs $K^*_{a/2,a/2}$, $K^*_{a/2,a/2} \setminus \{e\}$ or (c) $D^*[M_{y,x}] \in \Phi^m_a$, where $(a+1)/2 < m \leq a-1$.

(a). In this case, by Lemma 5 and Lemma 3(ii), D contains cycles of every length $2k$, $k \in [1, a]$.

(b). $D^*[M_{y,x}]$ is isomorphic to $K^*_{a/2,a/2}$ or $K^*_{a/2,a/2} \setminus \{e\}$ with partite sets $\{v_1, v_2, \ldots,$ $v_{a/2}\}$ and $\{v_{a/2+1}, v_{a/2+2}, \ldots, v_a\}$. Notice that $a \geq 4$ and $D^*[M_{y,x}]$ contains cycles of every length $2k$, $k \in [1, a/2]$. Therefore, by Lemma 3(ii), D contains cycles of every length $4k$, $k \in [1, a/2]$. It remains to show that for any $k \in [1, a/2-1]$, D also contains a cycle of length $4k+2$.

We claim that there exist $p \in [1, a/2]$ and $q \in [a/2+1, a]$ such that $y_px_q \in A(D)$. Assume that this is not the case, i.e., there is no arc from a vertex of $\{y_1, y_2, \ldots, y_{a/2}\}$ to a vertex of $\{x_{a/2+1}, x_{a/2+2}, ldots, x_a\}$. Then, since $D^*[M_{y,x}]$ is isomorphic to $K^*_{a/2,a/2}$ or $K^*_{a/2,a/2} \setminus \{e\}$, from the definition of $D^*[M_{y,x}]$ it follows that $d^+(y_1) \leq a/2$, $d^+(y_{a/2}) \leq a/2$, $d^-(y_1) \leq a/2+1$ and $d^-(y_{a/2}) \leq a/2+1$. Combining these inequalities, we obtain that $d(y_1)+d(y_{a/2}) \leq 2a+2$, which contradicts condition (A) since $a \geq 4$.

It suffices to consider the case when $D^*[M_{y,x}]$ is isomorphic to $K^*_{a/2,a/2} \setminus \{e\}$. Without loss of generality, we may assume that $e = v_a v_{a/2}$. From the definition of $D^*[M_{y,x}]$ it follows that $\{x_1, x_2, \ldots, x_{a/2}\} \to \{y_{a/2+1}, y_{a/2+2}, \ldots, y_a\}$ and D contains all possible arcs from $\{x_{a/2+1}, x_{a/2+2}, \ldots, x_a\}$ to $\{y_1, y_2, \ldots, y_{a/2}\}$ except $x_a y_{a/2}$.

If $p = a/2$ and $q = a$ (i.e., $y_{a/2}x_a \in A(D)$), then $y_1x_1y_{a/2+1}x_{a/2+1}y_2x_2y_{a/2+2}x_{a/2+2}\cdots y_kx_ky_{a/2+k}\, x_{a/2+k}y_{a/2}x_ay_1$ is a cycle of length $4k+2$, where $k \in [1, a/2-1]$. Thus, we may assume that $y_{a/2}x_a \notin A(D)$. Then the vertices x_a, $y_{a/2}$ are not adjacent since $x_ay_{a/2} \notin A(D)$. This together with $d^-(y_{a/2}, \{x_1, x_2, \ldots, x_{a/2}\}) \leq 1$ implies that $d(y_{a/2}) \leq 3a/2-1$. Therefore, by condition (A), $d(y_{a/2-1}) \geq 3a/2+1$ and hence, $y_{a/2-1}x_a \in A(D)$ since $d^-(y_{a/2-1}, \{x_1, x_2, \ldots, x_{a/2}\}) \leq 1$. Now it is not difficult to check that if $a \geq 6$, then $y_1x_1y_{a/2+1}x_{a/2+1}y_2x_2y_{a/2+2}x_{a/2+2}\,\cdots y_kx_ky_{a/2+k}x_{a/2+k}y_{a/2-1}x_ay_1$ is a cycle of length $4k+2$ when $k \in [1, a/2-2]$, and $y_1x_1y_{a/2+1}x_{a/2+1}y_2x_2\; y_{a/2+2}x_{a/2+2}\cdots x_{a/2-2}y_{a-2}x_{a-2}y_{a/2}x_{a/2}\, y_{a-1}x_{a-1}y_{a/2-1}\; x_ay_1$ is a cycle of length $2a-2$. If $a = 4$, then $y_2x_2y_4x_4y_1x_3y_2$ is a cycle of length $6 = 2a-2$.

(c). $D^*[M_{y,x}] \in \Phi^m_a$. Since D contains cycles of lengths 2, 4 (Lemma 5) and every digraph in Φ^m_a is Hamiltonian, we can assume that $a \geq 4$. Let $V(D^*[M_{y,x}]) = \{v_1, v_2, \ldots, v_a\}$ and $v_av_{a-1}\ldots v_2v_1v_a$ be a Hamiltonian cycle in $D^*[M_{y,x}]$. Therefore, by the definition of $D^*[M_{y,x}]$, for all $i \in [2, a]$, $x_iy_{i-1} \in A(D)$ and $x_1y_a \in A(D)$. From the definition of Φ^m_a we have $d^+(v_a) = 1$ and $d^+(v_{a-1}) \leq 2$. This means that $d^+(x_a) \leq 2$ and $d^+(x_{a-1}) \leq 3$. These together with $d^-(x_a) \leq a$, $d^-(x_{a-1}) \leq a$ and condition (A) implies that

$$d(x_a) \leq a+2, \quad d(x_{a-1}) \leq a+3 \quad \text{and} \quad 3a \leq d(x_a)+d(x_{a-1}) \leq 2a+5. \tag{13}$$

The last inequality of (13) implies that $a \leq 5$, i.e., $a = 4$ or $a = 5$.

Let $a = 5$. Then from (13) it follows that $d(x_a) + d(x_{a-1}) = 2a+5$, $d^-(x_a) = d^-(x_{a-1}) = a$, i.e., $\{y_1, y_2, \ldots, y_a\} \to \{x_a, x_{a-1}\}$. Therefore, $y_2x_5y_4x_4y_3x_3y_2$ (respectively, $y_1x_5y_4x_4y_3x_3y_2x_2y_1$) is a cycle of length 6 (respectively, of length 8).

Let $a = 4$. In this case, we need to show that D contains a cycle of length 6. If $x_1y_3 \in A(D)$ (or $y_2x_1 \in A(D)$), then $x_1y_3x_3y_2x_2y_1x_1$ (respectively, $x_1y_4x_4y_3x_3y_2x_1$) is a cycle of length 6. We may therefore assume that $x_1y_3 \notin A(D)$ and $y_2x_1 \notin A(D)$. Then $d(x_1) = d(x_4) = 6$ since $d(x_4) \leq a+2$, $d^+(x_a) \leq 2$ and $d(x_1) + d(x_4) \geq 12$. Therefore, $d^-(x_4) = 4$, which in turn implies that $y_1x_4 \in A(D)$. Hence, $y_1x_4y_3x_3y_2x_2y_1$ is a cycle of length 6. Thus, we have shown that if $D^*[M_{y,x}] \in \Phi^m_a$, then $a = 4$ or $a = 5$ and D contains cycles of all lengths $2, 4, \ldots, 2a$. This completes the proof of the theorem. □

5. Conclusion

In the current article, we prove a Meyniel-type condition and a Bang-Jensen, Gutin and Li-type condition for a strong balanced bipartite digraph of order $2a \geq 6$ to have cycles of all even lengths less than equal to $2a$.

It is worth to noting that over the past three years, various authors have received a number of sufficient conditions for the existence of cycles with certain properties in bipartite digraphs. In particular, several sufficient conditions for a balanced bipartite digraph to be

Hamiltonian or be even pancyclic were obtained (see, e.g., [23] by Wang and Wu, [24] by Adamus, [25] by Wang, [26] by Wang et al.).

A Hamiltonian path in a digraph D in which the initial vertex dominates the terminal vertex is called a Hamiltonian bypass in D. It was proved that a number of sufficient conditions for a digraph to be Hamiltonian is also sufficient for a digraph to contain a Hamiltonian bypass with some exceptions, which are characterized in [27], and the papers cited there. It is not difficult to show that, if a balanced bipartite digraph of order $2a \geq 4$ satisfies the conditions of Theorem 2(a) (or 2(b)), then D has a Hamiltonian bypass. In this regard, we believe that the the following conjecture is true.

Conjecture 3: D be a strong balanced bipartite digraph of order $2a \geq 6$. If D satisfies the conditions one of Theorems 2, 5 and 7, then D contains a Hamiltonian bypass, with some exceptions.

To conclude this section, we mention that Wang et al. [28] constructed an infinite family of counterexamples to Conjecture 2. Note that each of these counterexamples contains a vertex, which has degree equal to three.

Thus, Conjecture 2 remains open for digraphs with the minimum degree is at least four and for k-strong digraphs, where $k \geq 2$.

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Թեորեմ երկմասնյա կողմնորոշված գրաֆների զույգ համացիկլիկության մասին

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Ամփոփում

Ներկա աշխատանքում ապացուցվել է հետևյալ թեորեմը:

Թեորեմ: Դիցուք D-ն ուժեղ կապակցված $2a \geq 6$-գագաթանի երկմասնյա հավասարակշռված կողմնորոշված գրաֆ է, իսկ $\{x, y\}$-ը D գրաֆի գագաթների ցանկացած պարտվող կամ հաղթող զույգ է: Եթե տեղի ունի $d(x) + d(y) \geq 3a$ պայմանը, ապա D -ն պարունակում $2a$-ից փոքր կամ հավասար ցանկացած զույգ երկարության ցիկլ, բացի այն դեպքից, երբ D-ն $2a$ երկարության ցիկլ է:

Բանալի բառեր՝ Կողմնորոշված գրաֆ, համիլտոնյան ցիկլ, երկմասնյա կողմնորոշված գրաֆ, համացիկլիկ, զույգ համացիկլիկ:

Теорема о четных панциклических двудольных орграфах

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Аннотация

В настоящей работе доказана следующая теорема:

Теорема: Пусть D есть сильно связный $2a \geq 6$ - вершинный балансированный двудольный орграф. Предположим, что для каждой доминирующей и каждой доминирумой пары $\{x, y\}$ различных вершин имеет место $d(x) + d(y) \geq 3a$. Тогда D содержит контур любой четной длины $2k$, $0 \leq k \leq a$, кроме случая кокда D является контуром длины $2a$.

Ключевые слова: Орграф, гамильтонов цикл, двудольний орграф, панциклический орграф, четный панциклический орграф.

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Some Results on Palette Index of Cartesian Product Graphs

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Abstract

Given a proper edge coloring α of a graph G, we define the palette $S_G(v, \alpha)$ of a vertex $v \in V(G)$ as the set of all colors appearing on edges incident to v. The palette index $\check{s}(G)$ of G is the minimum number of distinct palettes occurring in a proper edge coloring of G. The windmill graph $Wd(n, k)$ is an undirected graph constructed for $k \geq 2$ and $n \geq 2$ by joining n copies of the complete graph K_k at a shared universal vertex. In this paper, we determine the bound on the palette index of Cartesian products of complete graphs and simple paths. We also consider the problem of determining the palette index of windmill graphs. In particular, we show that for any positive integers $n, k \geq 2$, $\check{s}(Wd(n, 2k)) = n + 1$.
Keywords: Edge coloring, Proper edge coloring, Palette, Palette index, Cartesian product, Windmill graph.
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1. Introduction

Throughout this paper, a graph G always means a finite undirected graph without loops, parallel edges, and it does not contain isolated vertices. Let $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. The degree of a vertex v in G is denoted by $d_G(v)$, and the maximum degree of vertices in G by $\Delta(G)$. The terms and concepts that we do not define can be found in [1].

An *edge coloring* of a graph G is an assignment of colors to the edges of G: it is *proper* if adjacent edges receive distinct colors. The minimum number of colors required in a proper edge coloring of a graph G is called the chromatic index of G and denoted by $\chi'(G)$. By Vizings theorem [9], the chromatic index of G equals either $\Delta(G)$ or $\Delta(G) + 1$. A graph with $\chi'(G) = \Delta(G)$ is called *Class 1*, while a graph with $\chi'(G) = \Delta(G) + 1$ is called *Class 2*.

In this paper, we consider a chromatic parameter called the palette index of a simple graph G. A proper edge-coloring of a graph defines at each vertex $v \in V(G)$ the set of colors of its incident edges. That set is called the *palette of* v and denoted by $S_G(v, \alpha)$. The minimum number of palettes, taken over all possible proper edge colorings of a graph G, is called a *palette index* of a graph and denoted by $\check{s}(G)$ [2]. Proper edge colorings with the minimum number of distinct palettes were studied for the first time in 2014, by Horňák, Kalinowski, Meszka, and Woźniak [2]. They determined the palette index of complete graphs. Namely,

$$\check{s}(K_n) = \begin{cases} 1, & \text{if n} \equiv 0(\text{mod}2) \\ 3, & \text{if n} \equiv 3(\text{mod}4) \\ 4, & \text{if n} \equiv 1(\text{mod}4) \end{cases} \tag{1}$$

Moreover, they also showed that the palette index of a d-regular graph is 1 if and only if the graph is of $Class$ 1. If G is d-regular and of $Class$ 2, then Vizings edge coloring theorem [9] implies that $3 \leq \check{s}(G) \leq d+1$, and the case $\check{s}(G) = 2$ is not possible, as proved in [2]. There are few results about the palette index of non-regular graphs. Vizings edge coloring theorem also yields an upper bound on the palette index of a graph G with maximum degree Δ and without isolated vertices, mainly $\check{s}(G) \leq 2^{\Delta+1} - 2$. In [6], Casselgren and Petrosyan provided an improvement and derived the following upper bound on the palette index of bipartite graphs:

$$\check{s}(G) \leq \sum_{d \in D_{even}(G)} \binom{\left\lceil \frac{\Delta(G)}{2} \right\rceil}{\frac{d}{2}} + \sum_{d \in D_{odd}(G)} \binom{\left\lceil \frac{\Delta(G)}{2} \right\rceil}{\frac{d+1}{2}} (d+1) \tag{2}$$

where $D_{odd}(G)$ is the set of all odd degrees in G and $D_{even}(G)$ is the set of even degrees in G.

In [3], Bonvicini and Mazzuoccolo proved that if G is 4-regular and of Class 2, then $\check{s}(G) \in \{3, 4, 5\}$, and that all these values are, in fact, attainable. Although it is possible to determine the exact value of the palette index for some classes of graphs, in general, it is an NP-complete problem, because from [4] it is known that computing the chromatic index of a given graph is an NP-complete problem.

In this paper, we provide upper and lower bounds on the palette index of Cartesian products of some graphs. We will give the exact number of palettes of $Wd(n, 2k)$ windmill graphs, as well as the upper and lower bounds for $Wd(n, 2k+1)$.

2. Preliminaries

In this section, we introduce some terminology and notation. A matching in a graph G is a set of pairwise independent edges of G. A matching that saturates all the vertices of G is called a perfect matching. Next, we need some additional definitions.

Definition 1: *(Windmill graph). The windmill graph $Wd(n, k)$ is an undirected graph constructed for $k \geq 2$ and $n \geq 2$ by joining n copies of the complete graph K_k at a shared universal vertex.*

Definition 2: *(Cartesian product of graphs). Let G and H be two graphs. The Cartesian product $G \square H$ of graphs G and H is a graph such that*

- *the vertex set of $G \square H$ is the Cartesian product $V(G) \times V(H)$.*
- *two vertices (u, u_1) and (v, v_1) are adjacent in $G \square H$ if and only if either*

– *$u = v$ and u_1 is adjacent to v_1 in H, or*

– *$u_1 = v_1$ and u is adjacent to v in G.*

Before we move on, we recall that the Cartesian product graph $G\square H$ decomposes into $|V(G)|$ copies of H and $|V(H)|$ copies of G. By the definition of Cartesian products of graphs, $G\square H$ has two types of edges: those the vertices of which have the same first coordinate, and those the vertices of which have the same second coordinate. The edges joining vertices with a given value of the first coordinate form a copy of H, so the edges of the first type form nH ($|V(G)| = n$). Similarly, the edges of the second type form mG ($|V(H)| = m$), and the union is $G\square H$.

Definition 3: *Given two graphs G and H, and a vertex $y \in V(H)$, the set $G^y = \{(x, y) \in V(G\square H) | x \in V(G)\}$ is called a G-fiber in the Cartesian product of G and H. For $x \in V(G)$, the H-fiber is defined as ${}^xH = \{(x, y) \in V(G\square H) \mid y \in V(H)\}$.*

G-fibers and H-fibers can be considered as induced subgraphs when appropriate. In [8], authors define the projection to G, which is the map $p_G : V(G\square H) \to V(G)$ is defined by $p_G(x, y) = x$. Also we will need the projection to H; $p_H : V(G\square H) \to V(H)$ is defined by $p_H(x, y) = y$.

In the proofs of our results, we also will follow some coloring ideas from [2]. Namely, we will use the coloring ideas described in the proofs of *Proposition 5*, which states that if $k \geq 0$, then $\check{s}(K_{4k+3}) = 3$, and *Theorem 7*, which shows that if $n = 4k + 5$, $k \neq 1$, then $\check{s}(K_n) = 4$.

3. Main Results

First, we will provide some results about the palette index of the Cartesian product of a cycle and simple path. Note that the palette index of $C_n\square P_2$ is equal to 1. Clearly, the Cartesian product of those graphs is a $Class$ 1 regular graph and as mentioned above the palette index of $Class$ 1 regular graph is equal to 1.

Proposition 1: *If $n = 2k$ and $m > 2$, then $\check{s}(C_n\square P_m) = 2$.*

Proof. First note that $C_n\square P_m$ is not a regular graph, hence, $\check{s}(C_n\square P_m) \geq 2$. Let construct a coloring that will induce 2 distinct palettes.

Case 1. m is even. Every $C_n - fiber$ can be properly colored alternately with colors a_1 and a_2. Because of the even length of cycles, we will get exactly one palette, denote it by $\{a_1, a_2\}$. Next, there are $n - pieces$ of $P_m - fibers$, and every $P_m - fiber$ can be properly colored alternately with colors a_3 and a_4. As a result, the palette of vertices with degree 3 is $\{a_1, a_2, a_3\}$, and the palette of vertices with degree 4 is $\{a_1, a_2, a_3, a_4\}$.

Case 2. m is odd. Suppose that $V(P_m) = \{v_1, v_2, ..., v_m\}$ and for any $i(1 \leq i \leq m - 1), v_iv_{i+1} \in E(P_m)$. Let $\alpha : E(C_n) \to \{a_1, a_2\}$ be a proper edge coloring of C_n. Since $C_n^{v_i}, 1 \leq i \leq m$ is isomorphic to C_n; hence, $C_n^{v_i}$ $(4 \leq i \leq m)$ can be properly colored with colors from the color-set $\{a_1, a_2\}$: $\forall(u, v_i), (u', v_i) \in V(C_n^{v_i})$ if $(u, v_i)(u', v_i) \in E(C_n\square P_m)$, then we define a proper edge coloring γ as follows:

$$\gamma((u, v_i)(u', v_i)) = \alpha(p_G(u, v_i)p_G(u', v_i)) = \alpha(uu') = a,$$

where $a \in \{a_1, a_2\}$. Afterwards, the fibers $C_n^{v_1}$, $C_n^{v_2}$ and $C_n^{v_3}$ can be colored alternately with colors from the color-sets $\{a_1, a_2\}$, $\{a_1, a_4\}$ and $\{a_1, a_3\}$, respectively. Then we will color the edges joining $C_n^{v_1}$ to $C_n^{v_2}$ and $C_n^{v_2}$ to $C_n^{v_3}$ by the colors a_3 and a_2, respectively. Observe that the remaining uncolored edges of $P_m - fibers$ can be properly colored alternately with colors a_3 and a_4; the obtained coloring γ is a proper edge coloring of $C_n \square P_m$ with a minimum number of palettes. ■

Using the same ideas makes it easy to obtain a coloring for $C_{2n+1} \square P_{2m}$, inducing 2 distinct palettes. When the number of vertices of the cycle and the number of vertices of the path are odd, we have the following theorem.

Theorem 1: *If* $n = 2k_1 + 1$ *and* $m = 2k_2 + 1$, $k_1, k_2 > 0$, *then*

$$\check{s}(C_n \square P_m) = 4.$$

Proof. Suppose that $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $d_{P_m}(v_1) = d_{P_m}(v_m) = 1$ and α is a coloring of $C_n \square P_m$ inducing $\check{s}(C_n \square P_m)$ distinct palettes. Let show that the value of the palette index is at least 4.

Case 1. $\check{s}(C_n \square P_m) = 1$. It follows that the graph is a regular graph, which is a contradiction.

Case 2. $\check{s}(C_n \square P_m) = 2$. Denote by P_1 and P_2 palettes induced by α. Clearly, $P_1 \cap P_2 \neq \emptyset$, therefore there is a color $a \in P_1 \cap P_2$ so that the edges colored with a form a perfect matching of the graph. However, $|V(C_n \square P_m)|$ is an odd number, which means that the graph cannot have a perfect matching, a contradiction.

Case 3. $\check{s}(C_n \square P_m) = 3$. Denote by P_1, P_2, and P_3 palettes induced by α. Suppose that $|P_1| = |P_2| = 3$ and $|P_3| = 4$. Clearly, there is no color belonging to all three palettes. Indeed, otherwise, that color would induce a perfect matching of $C_n \square P_m$, which is impossible. Assume that $(P_1 \cup P_2) \setminus P_3 \neq \emptyset$, then there is a color $a \in P_1 \cup P_2$ such that the edges colored with a form a perfect matching for C_n, which is impossible too, but this also means that the set $P_1 \cap P_2 \cap P_3$ cannot be empty, a contradiction.

Now, suppose that $|P_1| = |P_2| = 4$ and $|P_3| = 3$. Clearly there is a color $a \in P_1 \cap P_2$ and $a \notin P_3$. This implies that the edges colored with a form a perfect matching of $C_{n-2} \square P_m$, which is impossible. Hence, $\check{s}(C_n \square P_m) \geq 4$.

Next, we need to show the existence of a proper edge coloring α inducing four palettes. Assume that β is a proper edge coloring of C_n with colors from color-set $S = \{a_1, a_2, a_3\}$, inducing 3 distinct palettes. As we have already mentioned, $C_n^{v_i}, 1 \leq i \leq m-1$ can be properly colored with colors from the color-set S. Then for all $i (1 \leq i \leq m-1)$ the edge that joins $(u, v_i) \in V(C_n^{v_i})$ and $(u, v_{i+1}) \in V(C_n^{v_{i+1}})$ will be colored in one of the two ways, first if there is a color $a \in S$ that a does not belong to color-sets assigned to the incident edges of (u, v_i) and (u, v_{i+1}), then that edge will be colored with a. Otherwise it will be colored with a new color $a_4 \notin S$. Thereby we constructed coloring of the subgraph of $C_n \square P_m$, that is isomorphic to $C_n \square P_{m-1}$, inducing two palettes $\{a_1, a_2, a_3\}$ and $\{a_1, a_2, a_3, a_4\}$.

Note that the palette of the vertices of $C_n^{v_{m-1}}$ is $\{a_1, a_2, a_3\}$; hence, the colors assigned to the edges of $C_n^{v_{m-1}}$ divide that edge set into three disjoint sets: two sets X and Y, each having $\frac{n-1}{2}$ elements, and one one-element set, say $\{(u, v_{m-1})(u_1, v_{m-1})\}$. Without loss of generality, we may suppose that $(u, v_{m-1})(u_2, v_{m-1}) \in Y$ and X, Y are the sets of edges colored with a_1 and a_2, respectively. For all $(u', v_{m-1})(u'', v_{m-1}) \in X$ let do the following changes: $\alpha((u', v_{m-1})(u'', v_{m-1})) = a_4$, $\alpha((u', v_{m-1})(u', v_m)) = a_2$ and $\alpha((u'', v_{m-1})(u'', v_m)) = a_2$. Since (u, v_{m-1}) is the only vertex that the recent changes did not affect, $\alpha((u, v_{m-1})(u, v_m)) = a_4$. Finally, coloring the edge $\alpha((u, v_m)(u_1, v_m)) = a_5$ and the remaining uncolored edges alternately with colors a_2 and a_3 will induce two new palettes; hence, $\check{s}(C_n \square P_m) = 4$. ■

Next, we will examine the palette index of the Cartesian product of complete graphs and paths. Complete graph K_{2k} is of $Class$ 1 and $\check{s}(K_{2k}) = 1$, therefore $\check{s}(K_{2k} \square P_2) = 1$. On the other hand, the minimum coloring of K_{2k+1} induces $2k+1$ distinct palettes. Indeed, each palette has $2k$ colors. This means that exactly one color is missing at each vertex. So we can use the minimum coloring of K_n for all $K_n - fibers$ and color the edges joining them with missing colors, hence, $\check{s}(K_{2k+1} \square P_2) = 1$.

Corollary 1. *If $n > 2$ and $m > 2$, then $\check{s}(K_{2n} \square P_m) = 2$.*

Proof. Construction of a proper edge coloring of $K_{2n} \square P_m$ is very similar to the steps that we have already described in Proposition 1, the single difference being that in this case we will color $K_n - fibers$ with the minimum coloring described above. ■

Theorem 2: *For any odd positive integers m and $k \geq 0$, we have*

$$\check{s}(K_{4k+3} \square P_m) = 4.$$

Proof. Let $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $d_{P_m}(v_1) = d_{P_m}(v_m) = 1$. As we have already mentioned above there is a proper edge coloring with a minimum number of distinct palettes $\alpha : E(K_{4k+3}) \to S = \{a_1, a_2, a_3, ...a_{4k+3}\}$ inducing $4k+3$ different palettes. We will construct the coloring γ for $K_{4k+3} \square P_m$ as follows; $\forall i (1 \leq i \leq m-1)$ and $\forall (u, v_i)(u', v_i) \in E(K_{4k+3}^{v_i})$ $\gamma((u, v_i)(u', v_i))$ will be set equal to $\alpha(uu')$. Note that for any $i(1 \leq i \leq m-1)$, the vertices (u, v_i) and (u, v_{i+1}) are joined with the edges of $P_m - fibers$, and we have two possible cases for the coloring of these edges;

- if $S \setminus S_{K_{4k+3} \square P_m}((u, v_i)) = \{a\}$, then $\gamma((u, v_i)(u, v_{i+1})) = a$.
- if $S \setminus S_{K_{4k+3} \square P_m}((u, v_i)) = \emptyset$, then $\gamma((u, v_i)(u, v_{i+1})) = b$, $b \notin S$.

Note that the fiber $K_{4k+3}^{v_{m-1}}$ always has more than $k+1$ edges colored with the same color. Assume that $M = \{(u_{i_1}, v_{m-1})(u_{i_2}, v_{m-1}), ..., (u_{i_{2k+1}}, v_{m-1})(u_{i_{2k+2}}, v_{m-1})\}$ is the set of edges colored with $a' \in S$. Now let recolor some edges. For any $j(1 \leq j \leq k+1)$;

$$\gamma((u_{i_{2j-1}}, v_{m-1})(u_{i_{2j}}, v_{m-1})) = b,$$

$$\gamma((u_{i_s}, v_{m-1})(u_{i_s}, v_m)) = a', \ \forall s \in \{1, 2, ..., 2k+2\},$$

$$\gamma((u_i, v_{m-1})(u_i, v_m)) = b, \ \forall u_i \in V(K_{4k+3}^{v_{m-1}}) \setminus \{u_{i_1}, u_{i_2}, ..., u_{i_{2k+2}}\}$$

To color the edges of $K_{4k+3}^{v_m}$, we will follow the coloring idea introduced in the proof of [2](Proposition 5). Using the color-set $S \cup \{b_1, b_2, ...b_{2k+1}\} \cup \{b\}$ and taking the vertex set $X = \{u_{i_1}, u_{i_2}, ..., u_{i_{2k}}\}$, $Y = V(K_n^{v_m}) \setminus (X \cup \{u_{i_{2k+1}}\})$ and one-element set $\{u_{i_{2k+1}}\}$ will let us obtain coloring that induces 2 new palettes. Clearly, we can make the palette of the vertices from the vertex set X equal to $\{a_1, a_2, a_3, ...a_{4k+3}\}$, causing new palettes only on the vertices from the vertex set Y and $\{u_{i_{2k+1}}\}$; hence $\check{s}(K_{4k+3} \square P_m) \leq 4$.

Now let us show that the palette index is at least 4. Suppose first that $\check{s}(K_{4k+3} \square P_m) = 3$, and let α be the corresponding coloring of $K_{4k+3} \square P_m$. Denote by P_1, P_2 and P_3 the palettes caused by α. Let $V_i = \{x \in V : S(x,\alpha) = P_i\}, i = 1,2,3$. First, there is no color belonging to all three palettes, otherwise this color would induce a perfect matching of $K_{4k+3}\square P_m$, which is impossible.

Case 1. $|P_1| = |P_2| = n$, $|P_3| = n+1$. Note that $(P_1 \cup P_2) \setminus P_3 = \emptyset$; otherwise there is a color $a \in (P_1 \cup P_2) \setminus P_3$ then the edges colored with a form a perfect matching of K_n, which is impossible. It follows that $P_1 \cap P_2 \cap P_3 \neq \emptyset$, a contradiction.

Case 2. $|P_1| = |P_2| = n+1$, $|P_3| = n$. Clearly, there is an edge $e \in E(K_{4k+3}\square P_m,)$ joining V_1 and V_2. Assume that $\alpha(e) \notin P_3$, then the edges colored with $\alpha(e)$ will form a perfect matching of $K_{4k+1}\square P_m$, which is a contradiction.

Suppose next that $\check{s}(K_{4k+3}\square P_m) = 2$, the intersection of the induced palettes similarly cannot be an empty set. Hence, $\check{s}(K_{4k+3}\square P_m) \neq 2$. ■

Also, note that constructed coloring will induce at most 5 palettes for $K_{4k+5}\square P_{2m+1}$, the single difference being that in this case we will color the fiber $K_{4k+5}^{v_m}$ using the coloring constructed in the proof of [2](Theorem 7).

Corollary 2. *If $k \geq 0$ and $m \geq 1$, then*

$$4 \leq \check{s}(K_{4k+5}\square P_{2m+1}) \leq 5$$

Next results are about the palette index of windmill graphs.

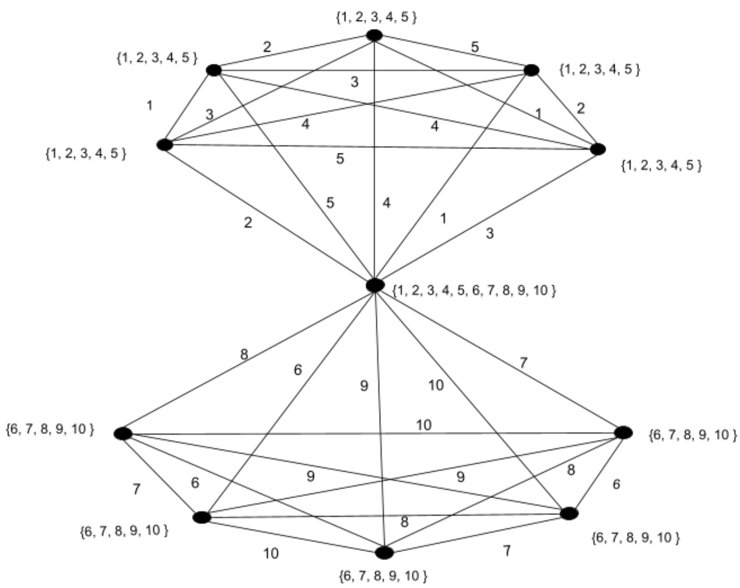

Fig. 1. $Wd(2,6)$ graph coloring.

Proposition 2: *If $n, k \geq 2$, then*

$$\check{s}(Wd(n,k)) \geq n+1.$$

Proof. Suppose that $\check{s}(Wd(n,k)) = m$ $(m < n+1)$. There is a proper edge coloring of $Wd(n,k)$ inducing m distinct palettes P_i, $i = 1,2,...,m$. Let V_i be the set of all vertices of K_n with palette P_i and let $n_i = |V_i|$, $i = 1,2,...,m$. Without loss of generality, suppose that $|V_m| = n_m = 1$ is a one-element set, say $\{u\}$. Assume that u is the shared vertex of $Wd(n,k)$. Clearly, $\sum_{i=1}^{m} n_i = |V(Wd(n,k))| = n(k-1)+1$. This implies that $\exists i(1 \le i \le m)$ that $n_i \ge k$. Indeed, if $n_i < k(1 \le i \le m)$, then it follows;

$$\sum_{i=1}^{m} n_i = \sum_{i=1}^{m-1} n_i + 1 \le (m-1)(k-1) + 1 < n(k-1) + 1 = |V(Wd(n,k))|,$$

which is impossible. Thus, $\exists j$ such that $|V_j| = n_j > k$. For any vertex of V_j there is an edge joining it with shared vertex u, and the number of such edges is equal to n_j. On the other hand, $n_j > |P_j| = k-1$, which is a contradiction; therefore $\check{s}(Wd(n,k)) \ge n+1$. ■

At the same time the upper bound of the palette index of windmill graphs depends on the number of complete graphs.

Theorem 3: *For any positive integers $n, k \ge 2$, we have $\check{s}(Wd(n,2k)) = n+1$.*

Proof. We only need to show the existence of a coloring α inducing $n+1$ palette. Denote by u the shared vertex of $Wd(n,2k)$. Note that $Wd(n,2k) - u$ is a graph that consists of n components, and every component is a complete graph with $2k-1$ vertices. For every K_{2k-1} complete graph exists coloring inducing $2k-1$ palettes, and at each vertex, exactly one color is missing, which will be assigned to the edge joining that vertex and the shared vertex u. Clearly, this coloring will induce exactly one palette on every odd component, and as a result, we will construct coloring α that will induce $n+1$ distinct palettes. ■
Fig.1 shows the proper edge coloring α of the graph $Wd(2,6)$ inducing 3 distinct palettes.

We will also give an upper bound for the palette index of $Wd(n,k)$ for any k odd number.

Corollary 3. *For any positive integers $k, n \ge 2$, we have*

$$\check{s}(Wd(n,k)) \le \begin{cases} 2n+1, & \text{if } k \equiv 3 \pmod 4, \\ 3n+1, & \text{if } k \equiv 1 \pmod 4. \end{cases} \tag{3}$$

Proof. Suppose that K_k^i $(1 \le i \le n)$ are the copies of the complete graph in $Wd(n,k)$, and u is the shared universal vertex. Denote by $C_1, C_2, ..., C_n$ disjoint color-sets needed for a proper edge coloring of a complete graph that induces a minimum number of distinct palettes.

Case 1. $k \equiv 3 \pmod 4$. We will use the coloring described in the proof of [2](Proposition 5). Assume that $\forall i(1 \le i \le n)$ α_i is a proper edge coloring of K_k^i with color-set C_i inducing 3 distinct palettes. While constructing the α_i coloring, the complete graph's vertex set is partitioned into three sets. One of these sets is a one-element set, which induces a new unique palette. Taking $\{u\}$ as that set for any partition of $V(K_k^i)(1 \le i \le n)$ will let us obtain coloring of $Wd(n,k)$ that induces at most $2n+1$ distinct palettes.

Case 2. $k \equiv 1 \pmod 4$. We will use the coloring described in the proof of [2](Theorem 7). Assume that $\forall i(1 \le i \le n)$ α_i is a proper edge coloring of K_k^i with color-set C_i inducing 4 distinct palettes. In this case, coloring α_i also causes a new unique palette on the vertex of one-element set. Similar to the previous case, taking $\{u\}$ as that set for all partitions of $V(K_k^i)$ $(1 \le i \le n)$ will let us obtain coloring of $Wd(n,k)$ that induces $3n+1$ distinct palettes. ■

4. Conclusion

In the current article we examined the palette index of Cartesian products of graphs. Namely, we determined the palette index of the Cartesian product of cycles and paths and constructed colorings based on the length of the cycle, inducing a minimum number of palettes. Next, we gave some results connected to the palette index of the Cartesian product of complete graphs and paths. We also considered the problem of determining the palette index of windmill graphs. In particular, we showed the existence of coloring α, such that the number of palettes of $Wd(n, 2k)$ for any $n, k \geq 2$ induced by α is equal to $n + 1$. Moreover, we determined the upper bounds for the windmill graphs in case when the number of vertices of each complete graph is odd.

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Որոշ արդյունքներ գրաֆների դեկարտյան արտադրյալների պալիտրայի ինդեքսի մասին

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Ամփոփում

Տրված է գրաֆի α-ճիշտ կողային ներկում, $S_G(v,\alpha)$-ով նշանակում են $v \in V(G)$ գագաթին կից կողերի բազմությունը: G ճիշտ կողային ներկման դեպքում իրարից տարբեր պալիտրաների նվազագույն քանակը անվանում են G- ի պալիտրային ինդեքս և նշանակում $\check{s}(G)$-ով: $Wd(n,k)$-ն չուղղորդված գրաֆ է, որը կառուցվում է $k \geq 2$ եվ $n \geq 2$ արժեքների համար n հատ K_k լրիվ գրաֆների մեկ ընդհանուր գագաթում միավորման միջոցով: Այս հոդվածում մենք կտանք պալիտրային ինդեքսի գնահատակն լրիվ գրաֆի եվ պարզ ճանապարհի դեկարդյան արտադրյալի համար: Մենք նաև տվել ենք պալիտրային ինդեքսի վերին գնահատականը $Wd(n,k)$-ի համար: Մասնավորապես, ցույց է տրվել, որ կամայական $k \geq 2$ եվ $n \geq 2$ զույգ ամբողջ թվերի համար $\check{s}(Wd(n,2k)) = n+1$:

Բանալի բառեր՝ Կողային ներկում, ճիշտ կողային ներկում, պալիտրա, պալիտրայի ինդեքս, Դեկարտյան արտադրյալ:

Некоторые результаты об индексе палитры декартово произведение графов

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Аннотация

При правильной α-реберной раскраске графа G мы определяем палитру $S_G(v,\alpha)$ вершины $v \in V(G)$ как множество всех цветов, появляющихся на ребрах, смежных с v Индекс палитры $\check{s}(G)$ графа G является минимальным числом различных палитр, встречающихся при всех правильных реберных раскрасках G. В теории графов мельница $Wd(n,k)$- это неориентированный граф, построенный для $k \geq 2$ и $n \leq 2$ путём предприятий n копии полных графов K_k в одной общей вершине В этой статье мы даем оценку индекса палитры декартового произведения полных графов и простых путей. Мы также рассматриваем задачу определения индекса палитры графов мельниц. В частности, мы показываем, что для любых положительных целых чисел $k \geq 2$ и $n \leq 2$, $\check{s}(Wd(n,2k)) = n+1$.

Ключевые слова: реберная раскраска, правильная рёберная, раскраска, палитра, индекс палитры, декартово произведение, граф мельница.

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The Role of Information Theory in the Field of Big Data Privacy

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Abstract

Protecting privacy in Big Data is a rapidly growing research area. The first approach towards privacy assurance was the anonymity method. However, recent research indicated that simply anonymized data sets can be easily attacked. Later, differential privacy was proposed, which proved to be the most promising approach. The trade-off between privacy and the usefulness of published data, as well as other problems, such as the availability of metrics to compare different ways of achieving anonymity, are in the realm of Information Theory. Although a number of review articles are available in literature, the information - theoretic methods capacities haven't been paid due attention. In the current article an overview of state-of-the-art methods from Information Theory to ensure privacy are provided.
Keywords: Big data, Anonymization, Differential privacy, Entropy, Mutual information, Distortion.
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1. Introduction

In recent years, Big Data has become a hot research topic, because it helps businesses and organizations to improve the decision making power and provides new opportunities with data analysis.

Big Data life cycle can be divided into the following stages: data generation, storage and processing. Multiple parties are involved in these stages, hence, the privacy violation risks are increased.

A number of privacy preserving mechanisms have been developed [1], [2], however, the study on Big Data privacy issues are at a very early stage [3]. Modern technologies and tools, such as social networks, search engines, hacking packages, data mining and machine learning tools, cause a lot of problems to individual privacy.

In general, it is very hard to find a clear definition or a global measurement on privacy. The studies on privacy can be separated into two classes: content privacy and interaction privacy. So far, the majority of research on privacy protection is conducted in the context of databases. The goal of a privacy preserving statistical database is to enable the user to learn properties of the population while securing the personal information.

The practically dominant privacy protection strategy is the use of cryptography. One way to protect data is to encrypt it in such a way that only the owner can decrypt it. The task of machine learning is to find the dependency in the data. An idea proceeds: why not to train the model on encrypted data? The problem with this approach is that when we encrypt data, the dependencies in it are lost, because this is the aim of encryption - to change the data so that the dependencies cannot be discovered. The degree of entropy in the data after encryption prevents models from capturing these dependencies. Therefore, encrypting data and training models on them do not work. The other disadvantage of cryptography for privacy is the limited computing power of mobile devices for safe encryption and decryption algorithms. That is why other methods for privacy preserving are required.

The main research categories of privacy are the data clustering and the theoretical frameworks. **Anonymization** is a key component of data clustering. Anonymization is the process of removing personal identifiers, both direct and indirect, that can lead to a person's identification. A person can be directly identified by name, address, zip code, telephone number, photograph or image, or other unique personal characteristics. A person can be indirectly identified if certain information is linked to other sources of information, including workplace, job title, salary, zip code, or even the fact of having a specific diagnosis or condition. Data cannot be completely anonymous and useful. Generally speaking, the richer the data, the more interesting and useful it is. This has led to the concepts of anonymization and removal of personally identifiable information, by which it is hoped that sensitive parts of the data can be suppressed to maintain the confidentiality of the records, while the rest can be published and used for analysis.

The early approach in data clustering direction is the k-anonymity method (1998), then its extension as l-diversity was suggested in 2007 and later the t-closeness method was developed in 2010. In the second category of privacy frameworks the differential privacy (DP) and its developments are included (Fig. 1). DP neutralizes linkage attacks, since it is a property of the data access mechanism and is not related to the presence or absence of auxiliary information available to the attacker.

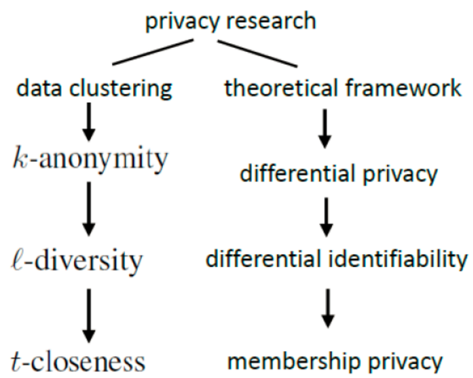

Fig. 1. The categories of privacy study [3].

When comparing syntactic and DP approaches, one can recall the trade-off between privacy and data utility. Finally, databases are passed on to provide certain benefits (for example, research knowledge, such as the effectiveness of a medical procedure). On the one hand, in order to maximize the usefulness of the data, all data can be published untouched, thus completely breaching privacy. On the other hand, not publishing any of the data will lead to maximum privacy, but this empty database will be useless. Therefore, the organization should seriously consider both its interests in data exchange and the risks they are willing to accept. An organization seeking to exchange sensitive data should consider the logistics issues involved in the exchange process and carefully balance the confidentiality and usefulness of the published data. An important step in one of these considerations is the choice of syntactic and DP. Such problems can be solved by information - theoretic methods and tools.

Although, there are some survey/review - type papers introducing the concept of privacy protection in Big Data, none of them provide detailed discussion regarding privacy with respect to Information Theory. In the current paper we introduce a summary of up-to-date methods on privacy assurance from Information Theory standpoints.

The rest of the paper is structured as follows. The DP is discussed in section 2. The developments of privacy based on information - theoretic methods are presented in section 3. The paper is summarized in section 4.

2. Differential Privacy

The DP framework was suggested in 2006 [4], that offers privacy protection in the sense of Information Theory. In recent years this topic has attracted attention and has been researched in literature. Main results, among many others, are surveyed in [5] - [8].

DP examines impossibility paradox of obtaining any information about a specific person by studying useful information about a multitude of people. Suppose the trusted party contains a set of sensitive personal data (eg email usage data, movie watching data, medical records) and wants to provide global statistical information about it. Such a system is called a statistical database. By providing such aggregated statistical information about the data, it is possible to disclose some information about individuals.

DP ensures that data about individuals from such a database cannot be retrieved, no matter what additional datasets or sources of information are available to the attacker. Such a guarantee is achieved due to the fact that the owner of the database uses such a mechanism (algorithm) for providing data, in which the presence or absence of information about a person in the database will not significantly affect the result of the request to it.

DP is designed to maximize the accuracy of queries from statistical databases while minimizing the possibility of disclosing the anonymity of records. The problem of analyzing sensitive data has a long history spanning many areas. As data about people become more and more detailed and technology allows more and more of this data to be collected and analyzed, there is an increasing need for a reliable, mathematically rigorous definition of privacy, as well as a class of algorithms that satisfy this definition. Various approaches to anonymizing data have failed when researchers have been able to identify personal information by combining two or more separate statistical databases. DP is the basis for the formalization of confidentiality in statistical databases and was introduced in order to protect against such methods of disclosing anonymity (deanonymization).

DP allows users to protect and maintain privacy when their data is in a specific database, just as they would be safe, if the data were not in some database. After the publication of human data in the database, in accordance with the differentiated confidentiality, the likelihood of violation of the confidentiality of people should not increase. That is, the degree of secrecy can be assessed by the likelihood of damage. This is one of the practical definitions of privacy.

DP can provide extremely strong guarantees of user privacy, but it does not guarantee unconditional relief from all damages. And it doesn't provide privacy where it didn't exist before. In general, DP does not guarantee that what a person considers his secret will remain secret. It simply ensures that participation in the survey is not disclosed by itself, that participation does not reveal any of the characteristics included in the survey. It is possible that the results of the survey may reflect statistical data about a person. Health screening for early signs of illness can produce strong, even convincing results. The fact that these findings are valid for humans does not imply a breach of confidentiality. The person may not even participate in the survey (DP ensures that these results are equally likely, regardless of whether the person participated in the survey or not).

It is desirable that DP be endowed with the following qualities: protection against arbitrary risks, automatic neutralization of linkage attacks, quantification of privacy loss.

DP is based on introducing randomness into data. To realize this, there are different mechanisms, e.g. the laplace mechanism, the exponential mechanism, mechanisms via α -nets, etc. Due to the fact that differential privacy is a probabilistic concept, any of its methods necessarily has a random component. Some of them, like Laplace's method, use the addition of controlled noise to the function to be calculated. Laplace's method adds Laplace noise, i.e. the noise from the Laplace distribution.

DP works by adding statistical noise to data (or its inputs or outputs). Depending on the location of the noise, DP is classified into two types: local DP and global DP (Fig. 2).

The most commonly used threat model in differential privacy is the **global DP model**. The main component is a trusted data curator. Each source sends him his confidential data, and it collects them in one place (for example, on a server). A repository is trusted if we assume that it processes our sensitive data on its own, does not transfer it to anyone, and cannot be compromised by anyone. In other words, we believe that a server with sensitive data cannot be hacked. Within the central model, we usually add noise to query responses. The advantage of this model is the ability to add the lowest possible noise value, thus maintaining the maximum accuracy allowed by the principles of DP. The disadvantage of the central model is that it requires a trusted store, and many of them are not. In fact, the lack of trust in the consumer of the data is usually the main reason for using DP principles.

The **local DP model** allows you to get rid of the trusted data store: each data source (or data owner) adds noise to their data before transferring it to the store. This means that the storage will never contain sensitive information, implying there is no need for its power of attorney. The local model of DP avoids the main problem of the central model: if the data warehouse is compromised, then hackers will only have access to noisy data that already meets the requirements of DP.

The local model is less accurate than the central one. In the local model, each source independently adds noise to satisfy its own differential privacy conditions, so that the total noise from all participants is much greater than the noise in the central model. Ultimately, this approach is only justified for queries with a very persistent trend (signal). Apple, for example, uses a local model to estimate the popularity of emoji, but the result is only useful for the most popular emoji (where the trend is most pronounced). Typically, this model is not used for more complex queries, such as those used by the US Census Bureau or machine learning. The central and local models have both advantages and disadvantages, and now

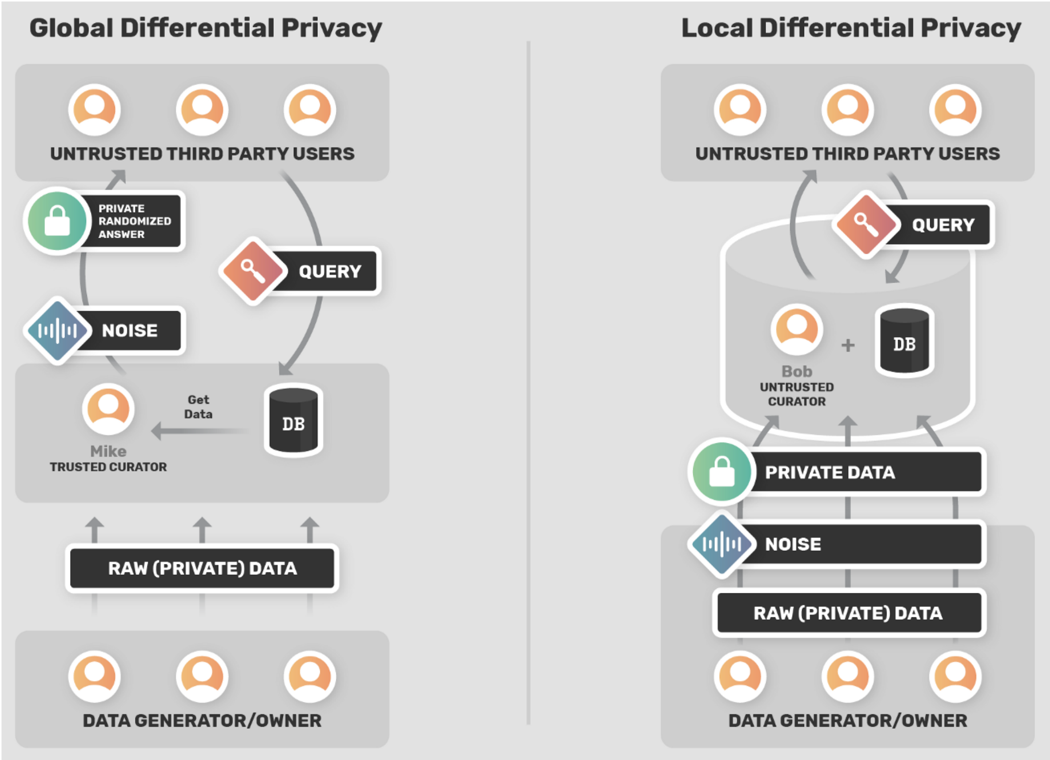

Fig. 2 Global Differential Privacy and Local Differential Privacy

the main effort is to get the best of them.

3. Review of Information Theory - based Results in Privacy

The first problem is to have a metric to compare various ways of achieving anonymity. The initial focus on analyzing the anonymity of messaging through mixed - based anonymity systems in which all network communication is available for the attacker is given in [9]. An information - theoretic metric based on the idea of anonymity probability distributions is introduced. In the same paper it is demonstrated that if maximum route length in the mix system exists, it is known to the attacker and can be used to extract additional information. It means that more advanced probabilistic metrics of anonymity are needed.

An analytical measure of anonymity of routs in eavesdropped networks is proposed in [10] using the information-theoretic equivocation. Cryptographic techniques prevent analysis of packet content, however, information can be gained by analyzing the correlation of transmission schedules of multiple nodes, as the packet timing information is easy to obtain in wireless networks. For anonymity it is necessary for the routes to be undetectable using the correlation across the transmission schedules, which results in a tradeoff between anonymity and network performance. For this purpose a quantifiable metric is defined in [10] using the uncertainty in networking information. The key result shows the equivalence between anonymity - performance tradeoff and information - theoretic rate distortion.

It is often important to allow researchers to analyze data without compromising the privacy of individuals or leaking confidential information outside the organization. In [11] it is shown that sparse regression for high dimensional data can be carried out directly on a compressed form of the data, in a manner to guard privacy in information - theoretic sense.

The suggested compression reduces the number of data records exponentially preserving the number of input variables. These compressed data can then be made available for statistical analyses with the same accuracy as the original data. In this case the original data are not recoverable from the compressed data, and the algorithms run faster, requiring fewer resources. The privacy (the problem of recovering the uncompressed data from the compressed one) is evaluated in information - theoretic terms by bounding the average mutual information, which is connected with the problem of computing the channel capacity of certain systems.

A new privacy measure in terms of Information Theory, similar to t-closeness is defined in [12], which can be achieved by the postrandomization method in the descrete case and by noise addition in the general case. The privacy criterion here is an average measure over a divergence, and the privacy - distortion problem is strongly related to the rate - distortion problem in the field of Information Theory, namely, the problem of lossy compression of source data subject to a distortion criterion.

A new measure for privacy of votes is proposed in [13], that relies on the notion of entropy. Entropy is a natural choice to measure privacy in an information - theoretic setting, and authors demonstrate how different formulations of conditional entropy answer different questions about vote privacy. A theorem has been established that enables accurate analysis of privacy offered by complex cryptographic voting protocols. Connections between two existing privacy notions for votes have been established.

The study of DP from a rate - distortion perspective has been initiated in [14]. Rate - distortion is applicable when the goal of the data collector is to publish an approximation of the data itself. The case when the data collector is not trusted is considered, which leads to using the local DP as a privacy measure. A robust rate-distortion setting is considered, in which the source distribution is unknown, but comes from some class. The goal is to look for a locally differentially private channel, that achieves minimum privacy risks while guaranteeing distortion of the given level.

In [15] the relation between three different notions of privacy: identifiability, differential privacy and mutual - information privacy is investigated. Identifiability guarantees indistinguishability between probabilities, DP guarantees limited additional disclosures, and mutual information is the information - theoretic notion. Under a unified privacy - distortion framework, where the distortion is the Hamming distance between the input and output databases, some connections between these three privacy notions have been established.

Guaranteeing a tight bound on privacy risk often incurs a significant penalty in terms of the usefulness of the published result. This privacy-utility tradeoff is studied in [16] in the context of publishing a differentially private approximation of the full data set and measure utility via a distortion measure.

4. Conclusion

In this article a general outlook on the current methods for estimating privacy of databases from Information Theory perspectives is provided. A series of publications devoted to various problems of privacy solved by information - theoretic tools and methods is analyzed. Research has shown that information-theoretic methods are effective for a wide range of tasks ranging from anonymity to differential privacy.

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Ինֆորմացիայի տեսության դերը մեծ տվյալների գաղտնիության ոլորտում

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Ամփոփում

Մեծ տվյալների հետ աշխատելիս գաղտնիության պահպանումը հետազոտության արագ աճող ոլորտ է: Գաղտնիության առաջին մոտեցումը անանունության մեթոդն էր: Վերջին ուսումնասիրությունները ցույց են տվել, որ պարզապես անանուն տվյալների շտեմարանները կարող են հեշտությամբ ենթարկվել հարձակման գաղտնիության տեսանկյունից: Հետագայում առաջարկվեց դիֆերենցիալ գաղտնիությունը, որն ապացուցեց, որ ամենահեռանկարայինն է: Գաղտնիության և հրապարակված տվյալների օգտակարության միջև փոխզիջումը, ինչպես նաև այլ հարցերի, ինչպիսիք են անանունության հասնելու տարբեր եղանակները համեմատելու համար չափի առկայությունը, ընկնում է ինֆորմացիայի տեսության տիրույթում: Չնայած գրականության մեջ մի շարք ակնարկային հոդվածների առկայությանը, ինֆորմացիայի տեսության մեթոդների հնարավորությունները պատշաճ ուշադրության չեն արժանացել: Այս հոդվածում մենք ներկայացնում ենք ինֆորմացիայի տեսության ժամանակակից մեթոդների ակնարկ՝ գաղտնիությունն ապահովելու համար: Վերլուծվում են ինֆորմացիայի տեսության գործիքների և մեթոդների միջոցով լուծված գաղտնիության տարաբնույթ խնդիրներին նվիրված մի շարք հրապարակումներ: Հետազոտությունները ցույց են տվել, որ ինֆորմացիոն տեսական մեթոդներն արդյունավետ են խնդիրների լայն շրջանակի համար՝ անանունությունից մինչև դիֆերենցիալ գաղտնիություն:

Բանալի բառեր՝ Մեծ տվյալներ, անանունացում, դիֆերենցիալ գաղտնիություն, Էնտրոպիա, փոխադարձ ինֆորմացիա, շեղում:

Роль теории информации в области конфиденциальности больших данных

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Аннотация

Защита конфиденциальности при работе с большими данными - быстрорастущая область исследований. Первым подходом к конфиденциальности был метод анонимности. Недавние исследования показали, что просто анонимные наборы данных могут быть легко атакованы с точки зрения конфиденциальности. Позже была предложена дифференциальная конфиденциальность, которая оказалась наиболее многообещающей. Компромисс между конфиденциальностью и полезностью опубликованных данных, а также другие проблемы, такие как наличие метрик для сравнения различных способов достижения анонимности, относятся к сфере теории информации. Несмотря на наличие в литературе ряда обзорных статей, возможностям методов теории информации не уделялось должного внимания. В этой статье мы даем обзор новейших методов теории информации для обеспечения конфиденциальности. Анализируется серия публикаций, посвященных различным проблемам конфиденциальности, решаемым с помощью инструментов и методов теории информации. Исследования показали, что теоретико-информационные методы эффективны для широкого круга задач, от анонимности до дифференциальной конфиденциальности.

Ключевые слова: Большие данные, анонимизация, дифференциальная конфиденциальность, энтропия, взаимная информация, искажение

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Application of Deep Learning-Based Methods to the Single Image Non-Uniform Blind Motion Deblurring Problem

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Abstract

In this paper, we present deep learning-based blind image deblurring methods for estimating and removing a non-uniform motion blur from a single blurry image. We propose two fully convolutional neural networks (CNN) for solving the problem. The networks are trained end-to-end to reconstruct the latent sharp image directly from the given single blurry image without estimating and making any assumptions on the blur kernel, its uniformity, and noise. We demonstrate the performance of the proposed models and show that our approaches can effectively estimate and remove complex non-uniform motion blur from a single blurry image.

Keywords: Motion blur, Blind motion deblurring, Non-uniform blurring, Blur kernel.

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1. Introduction

Motion blur is one of the most undesired types of image degradation when taking photos. The shake of the camera and the object motion during the exposure cause motion blurry images. Motion blur is an undesirable effect, particularly in photography, and still is considered an effect, which causes a significant distortion of an image. The process of recovering the latent sharp image from a single motion blurry image or from a sequence of blurry video frames is called motion deblurring. In practice, there are a large number of possible motion paths, and every motion-blurred image is uniquely blurred, thus motion deblurring is a common and challenging problem nowadays.
A high-level representation of the blurring process is the following model

$$b = I \otimes \mathrm{f} + \mathrm{n}, \quad (1)$$

where I is the latent sharp image, f is the blur kernel, n denotes the noise, and $\otimes$ is the convolution operator. In the presence of only one blurry image, the problem is called single-image motion deblurring. In the case of multiple sequential blurry images, the problem is called multi-image/video motion deblurring. Our interest is mainly related to single-image motion deblurring.

If the blur kernel or point spread function (PSF) is shift-invariant in the sense that blurring is uniform, then the deblurring problem turns into the image deconvolution problem. When the point spread function (PSF) is shift-variant and therefore the blurring is non-uniform, then it is considered a deblurring problem.

Image deblurring is categorized as non-blind and blind cases. In the case of non-blind deblurring, the blur kernel is known, or there is a way to compute it using some prior knowledge, so the problem turns to estimate the latent sharp image given the known blur kernel. There are some difficulties to overcome even though it may seem not a hard task. For example, the presence of noise and possible ringing artifacts arising during deblurring make it a challenging problem.

There are some traditional methods such as Wiener deconvolution [1] which is expressed as

$$G(f) = \frac{H^*(f)\, S(f)}{|H(f)|^2\, S(f) + N(f)}, \quad (2)$$

where f is the frequency in the frequency domain, G is the Fourier transform of the estimated kernel, which then is convolved with the blurry image to estimate the latent sharp image, H is the Fourier transform of the blur kernel, N and S are the mean power spectral density of the noise and latent sharp image respectively, $*$ denotes the complex conjugation. Iterative Richardson-Lucy (RL) [2, 3] deconvolution is another method, which is expressed as

$$I^{t+1} = I^t \left(PSF^T \otimes \left(\frac{B}{I^t \otimes PSF}\right)\right), \quad (3)$$

where I^t and I^{t+1} are t^{th} and $(\mathrm{t}+1)^{\mathrm{th}}$ estimations of the latent sharp image I, B is the blurry image and PSF^T is the flipped version of PSF.

These methods were presented decades ago. In further studies, the solution to the problem of non-blind deblurring tends to be based on many famous image priors, for example, sparse priors [4] and total variation [5], which have been introduced for regularization purposes to improve the quality of deconvolution in the presence of noise.

The blind deblurring [6] is a more challenging problem since in this case the blur kernel or PSF is also unknown in addition to the unknown latent sharp image. The blind deblurring problem consists of two stages: the PSF estimation and non-blind deconvolution. In contrast to non-blind deblurring, more sophisticated priors have been introduced here, such as norm-based prior [7], dark channel prior [8], reweighted graph total variation prior [9], etc.

Image deblurring methods are also categorized as deep learning-based (DL) and non-deep learning-based (non-DL) or optimization-based methods. Non-DL-based or optimization-based methods try to reconstruct the latent sharp image by minimizing the energy function [10, 11], using, for example, Gaussian or Poisson likelihoods in the scope of maximum-a-posteriori estimation [12].

Even though non-DL-based methods are effective in image deblurring, they are usually based on relatively simplified assumptions on the blur model compared with DL-based methods. It is also worth mentioning the time-consuming hyperparameter tuning process for non-DL-based methods, which is significant in real-world cases. In recent years, DL-based approaches have become more and more applicable. DL-based methods use convolutional neural networks to reconstruct the latent sharp image [13]. Also, recurrent neural networks are used for single image deblurring [14]. In terms of both accuracy and efficiency, these methods exceed non-DL methods.

So, we present deep learning-based blind image deblurring methods for estimating and removing non-uniform motion blur from a single blurry image.

2. Dataset

A common practice for creating a dataset for supervised image deblur problems is to synthetically generate blurry images by blurring latent sharp images with a kernel and then adding some noise [15, 16]. However, the blurry images generated in this way may differ from a real blurry image, and the dataset might not be representative enough.

A new kernel-free approach of dataset generation for supervised motion deblur problems was proposed in [17]. They used a GOPRO4 Hero Black camera for dataset generation. They record high-quality videos with 240 fps and then average sequential video frames of latent sharp images to produce motion blurry images [18]. The corresponding latent sharp image for the generated blurry image is chosen as the middle image of the sequence that is used to average and generate the blurry image.

When the motion blur is caused by the motion of an object, the blurriest part of the blurry image should be the object itself, leaving the background mostly the same as in the latent sharp image. The proposed kernel-free dataset generation method [17] for supervised motion deblur problems solves that problem unlike the other methods [15, 16].

We chose the GOPRO dataset [18] for training and evaluating our models. The dataset contains 3214 pairs of blurry and sharp images.

3. Proposed Methods

We propose two encoder-decoder architecture based fully convolutional neural networks.

The first one (ResnetEncDec) uses Resnet-50 [19] as an encoder. It receives a 3x256x256 RGB image as input. The first step is a convolution with a 7x7 kernel with stride 2 followed by max-pooling with stride 2. Then the Resnet-50 residual blocks follow, which use 1x1 and 3x3 convolutions. Each convolution layer is followed by a batch normalization layer [20] and ReLU activation. The encoder part outputs a 2048x8x8 feature map, which is used as an input of the decoder part.

The decoder part consists of transposed convolution and upsample layers. First, 3 decoder blocks follow, each of which consists of a transpose convolution layer followed by 2 convolutions. Then, 2 upsample layers follow, each of which performs a bilinear upsampling with a factor of 2 followed by 2 convolutions. Then, a 1x1 convolution follow to reduce the channels of the activation map to 3. Then, a sigmoid activation follow to output colors in [0, 1] range for each pixel of the output image. All the convolution and deconvolution layers are followed by batch normalization and ReLU activation (except the last convolution layer, which is followed by sigmoid activation).

The skip connections are used between the encoder and decoder layers inspired by the U-Net architecture [21]. The architecture of the network is shown in Figure 1.

The next proposed network is inspired by the real-time style transfer method proposed in [22]. They propose using an image transform network (TransformNet) for the style transfer problem to stylize the input content image with the style of the style image (Fig 2). Since the network performed well on style transfer image to image problem, thus, being able to generate an image that is some modified version of the input image, we proposed it for the motion deblur problem.

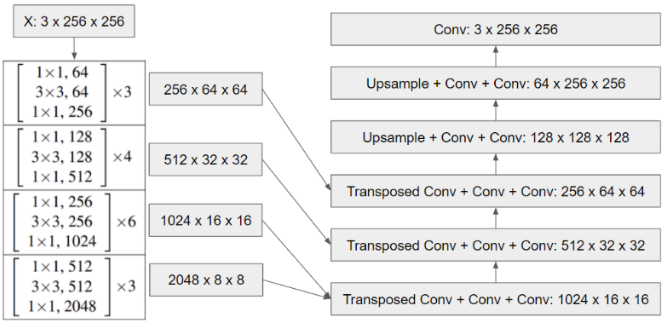


Fig. 1. The architecture of the ResnetEncDec fully convolutional network.

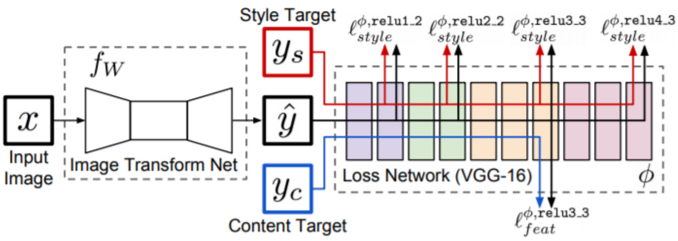
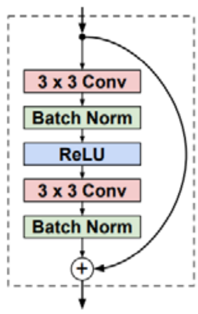


Fig. 2. The architecture of the style transfer network [22].

The first layer of the proposed Transform Net is a 9x9 convolution with stride 1. Then two 3x3 convolutions follow with stride 2. Then, 5 residual blocks follow, each of which consists of two 3x3 convolutions followed by batch normalization and ReLU activation (Fig. 3). Each residual block contains a residual connection between its input and output. After the 5 residual blocks, two 3x3 transposed convolution layers follow with stride 2. Then, a 9x9 convolution follow with stride 1. Finally, sigmoid activation follows to output colors in [0, 1] range for each pixel of the output image. Each convolution layer is followed by batch normalization and ReLU activation (except the last convolution layer, which is followed by sigmoid activation).

Layer	Activation size
Input	$3 \times 256 \times 256$
$32 \times 9 \times 9$ conv, stride 1	$32 \times 256 \times 256$
$64 \times 3 \times 3$ conv, stride 2	$64 \times 128 \times 128$
$128 \times 3 \times 3$ conv, stride 2	$128 \times 64 \times 64$
Residual block, 128 filters	$128 \times 64 \times 64$
Residual block, 128 filters	$128 \times 64 \times 64$
Residual block, 128 filters	$128 \times 64 \times 64$
Residual block, 128 filters	$128 \times 64 \times 64$
Residual block, 128 filters	$128 \times 64 \times 64$
$64 \times 3 \times 3$ conv, stride 1/2	$64 \times 128 \times 128$
$32 \times 3 \times 3$ conv, stride 1/2	$32 \times 256 \times 256$
$3 \times 9 \times 9$ conv, stride 1	$3 \times 256 \times 256$

(a)



(b)

Fig. 3. (a) The architecture of the TransformNet. [23] (b) The architecture of each residual block [23].

4. Training

Both proposed networks are trained on the GOPRO dataset with 256x256 resized images. Since we want to minimize the pixel-wise differences between the output and latent sharp image in the motion deblur problem, we chose MSE [24] and MAE [25] as loss functions:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2, \tag{4}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|\hat{y}_i - y_i|, \tag{5}$$

where N is the number of pixels in the image, y is the pixel value of the sharp image and $\hat{y}$ is the predicted pixel value.

Our experiments showed that MSE performs better for both of the networks, at least at the early steps of training, so we used MSE for further experiments.
As evaluation metrics we chose PSNR (peak signal-to-noise ratio) [26] and MSE functions:

$$PSNR = 20\log_{10}\left(\frac{MAX_i}{\sqrt{MSE}}\right), \tag{6}$$

where MAX_i is the maximum possible pixel value of the image.

The Adam optimizer [27] was used with a learning rate of 0.001. Both networks are trained for 350 epochs with batch sizes 15 and 44 for ResnetEncDec and TransformNet correspondingly running on GeForce GTX 1070 Ti GPU. ImageNet [19] pre-trained weights are used to initialize the ResnetEncDec encoder part. For TransformNet, training continued additionally for 250 epochs with SGD optimizer [28] without momentum with a learning rate of 0.0001. However, it does not lead to significant improvements.

The learning curves of both networks are shown in Figure 4.

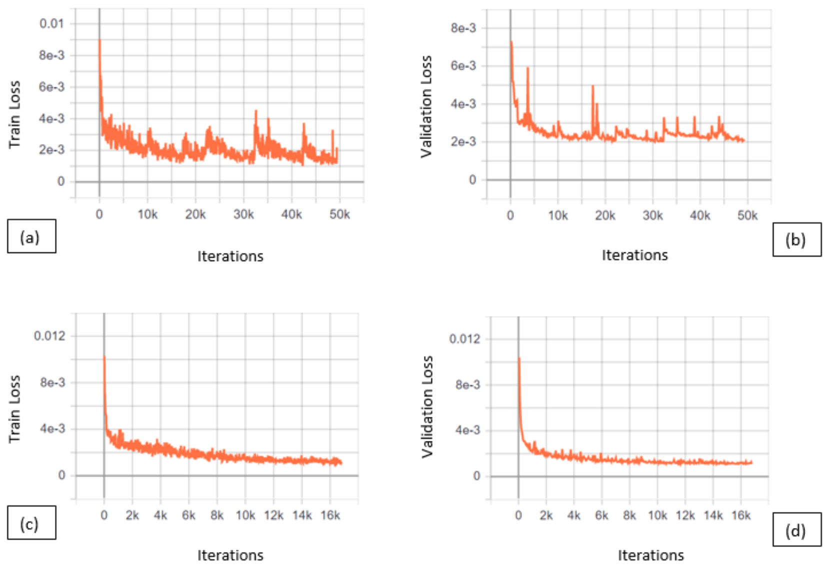


Fig 4. The learning curves of ResnetEncDec (a, b) and TransformNet(c, d).

5. Results

We evaluate the performance of our proposed models on the GOPRO dataset. The results are compared with one of the state-of-the-art methods [17]. The quantitative performance comparison of the proposed models is shown in Table 1 (note that we use 256x256 resized images, while in [17] they use images with an original size of 1280x720).

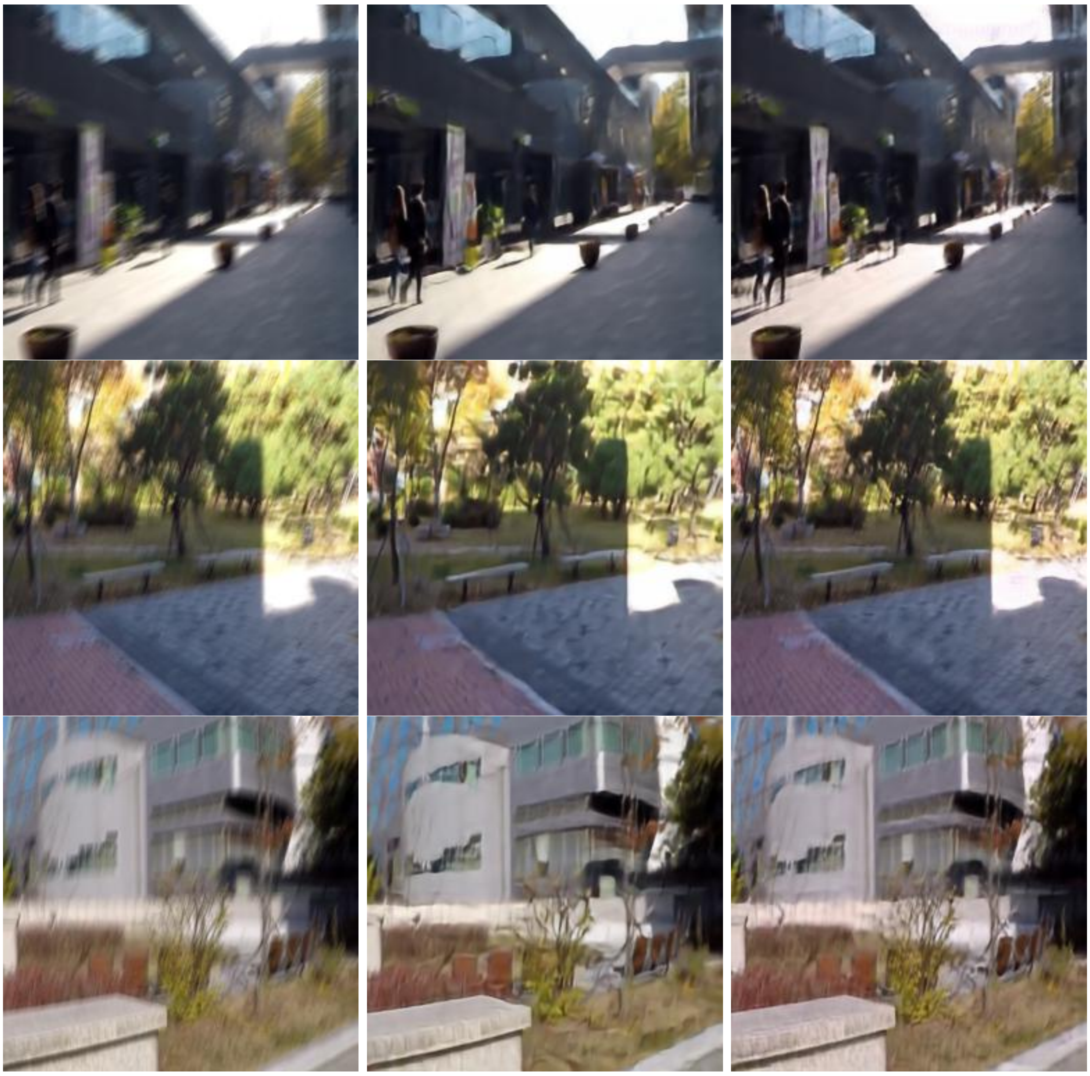
Table 1: Quantitative performance comparison of the models.

Metrics	*ResNetEncDec*	*TransformNet*	*Nah et al. [17]*
PSNR	24.98	26.26	28.93
MSE	0.0033	0.00245	-

Some deblurring results are shown in Fig. 5.

In terms of performance and memory usage, the TransformNet and ResNetEncDec are lightweight networks compared to [17], since [17] relies on a deep multi-scale architecture.

At the same time, as it is obvious from the architectures of the proposed networks, the TransformNet is more lightweight and requires less computational time and resources than the ResNetEncDec.



Input image ResnetEncDec result TransformNet result

Fig 5. The results on GOPRO test dataset.

6. Conclusion

In this paper, two deep learning-based blind motion deblurring methods were presented to reconstruct the latent sharp image from a single motion blurry image without having any information about the blur kernel, its uniformity, and existing noise. The proposed methods, which are encoder-decoder architecture-based fully convolutional neural networks, were trained, validated and evaluated on the GOPRO dataset [18] (using 256x256 resized images) and compared with one of the state-of-the-art methods presented in [17]. Based on the results shown in Table 1 and Figure 5, it becomes clear that the proposed methods can effectively remove complex non-uniform motion blur demonstrating acceptable results. The code and results are available at https://github.com/Mekhak/motion_deblur_dl.

Future work should address improving the accuracy of the proposed methods.

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Խորը ուսուցման վրա հիմնված մեթոդների կիրառումը պատկերում շարժման հետևանքով առաջացած ոչ միատարր պղտորման հեռացման կույր խնդրում

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Ամփոփում

Հոդվածում ներկայացվում են խորը ուսուցման վրա հիմնված պատկերից պղտորման հեռացման կույր մեթոդներ՝ պատկերում շարժման հետևանքով առաջացած ոչ միատարր պղտորման գնահատման և հեռացման համար: Խնդրի լուծման համար առաջարկվում են երկու լրիվ փաթույթային նեյրոնային ցանցեր (CNN): Տրված պղտորված պատկերից սկզբնական սուր պատկերը վերականգնելու համար ներկայացված ցանցերը ուսուցանվում են ամբողջապես՝ առանց գնահատելու և որևէ ենթադրություններ անելու պղտորման միջուկի, նրա միատարրության և առկա աղմուկի վերաբերյալ: Ցուցադրվում է առաջարկվող մոդելների արտադրողականությունը և ցույց է տրվում, որ առաջարկվող մոտեցումները կարող են արդյունավետորեն գնահատել և հեռացնել շարժման հետևանքով պատկերում առաջացած բարդ, ոչ միատարր պղտորումը:

Բանալի բառեր՝ Շարժման հետևանքով առաջացած պղտորում, շարժման հետևանքով առաջացած պղտորման հեռացում, ոչ միատարր պղտորում, պղտորման միջուկ:

Применение методов глубокого обучения в задаче слепого устранения размытости вслед за движением из одного неоднородно размытого изображения

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Аннотация

В этой статье представляются слепые методы устранения размытости изображения основанные на глубоком обучении – для оценки и удаления неоднородного размытия вслед за движением из одного размытого изображения. Для решения задачи предлагаются две полностью сверточные нейронные сети (CNN). Сети, предназначенные для восстановления исходного резкого изображения из размытого изображения, обучаются полностью – без оценки и каких-либо предположений о кернеле размытия, его однородности и присутствующего шума. Демонстрируется производительность предложенных моделей и показано, что предложенные методы могут эффективно оценивать и устранить сложное неоднородное размытие вслед за движением из одного размытого изображения.

Ключевые слова: Размытие из за движения, слепое устранение размытости вслед за движением, неоднородное размытие, кернел размытия.

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Designing and Implementing a Method of Data Augmentation Using Machine Learning

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Abstract

Efficiency of neural network (NN) models depend on the parameters given and the input data. Due to the complexity of environmental conditions and limitations the data for NN models, especially for the case of images, can be insufficient. To overcome this problem data augmentation has been used to enlarge the dataset. The task is to generate diverse set of images from a small set of images for NN training. Due to data augmentation transformation, 3105 new images out of 345 input data were created for classification, detection and image segmentation.

Keywords: Machine Learning, Convolutional Neural Networks, Data Augmentation, burn degrees.

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1. Introduction

Nowadays image classification problems are mainly solved via convolutional neural networks (CNN). Deep learning CNN needs a huge number of images for the model to be trained effectively. If the issue of data scarcity is faced, the simple, yet effective techniques such as transformations may pose a limited solution [1, 2].

Data augmentation techniques in data analysis are used to generate slightly modified copies or create synthetic data from a real dataset and hence artificially increase its size. CNN is known to be invariant, meaning that it can robustly classify objects with different transformations. Hence, the increase of relevant data in the dataset will result in a better accuracy for the CNN model.

Besides adding more images to our dataset, the data augmentation tools are beneficial for having images in a limitless set of conditions. In real-world scenarios pictures can be taken in different orientation, location, scale, brightness, etc. Therefore, various transformations of the same image such as rotation and cropping, can enhance the utility of the training set and increase the performance of ML algorithms.

There are two options for data augmentation:

- online augmentation or augmentation on the fly is applied in real time. This method performs transformations on the mini-batches and then fits into the model. This means that the online augmentation will see different images at each epoch. This kind of expansion is preferred for larger datasets, otherwise there would be an explosive increase in size.
- offline version transforms each image in the training set by rotating, cropping, etc. As a result, the size of the training dataset increases by a factor equal to the number of transformations performed on the image. Offline data augmentation is preferred for relatively smaller datasets as the goal is having more images to train for the model [3, 4].

2. Description of the Dataset

The collected dataset consists of around 400 images of human skin burns. The pictures of the burns are taken from different angles mainly in a monotone background. The dataset is labeled into three classes according to their burn degrees, examples of which are presented in Fig. 1.

Fig.1. Some examples of images from the dataset

Before designing the model, the dataset is divided into two sets: training and test. On the first one the model is trained, while the second one is used to determine the accuracy of the designed model. In our case 345 out of 400 images form the training dataset. As the dataset was initially labeled into 3 different classes, the training and test sets should consist of items of three classes with each class containing nearly equal number of images. In our case we have approximately 120 images in each class for the training set and 30 images in each class for the test set (Fig. 2).

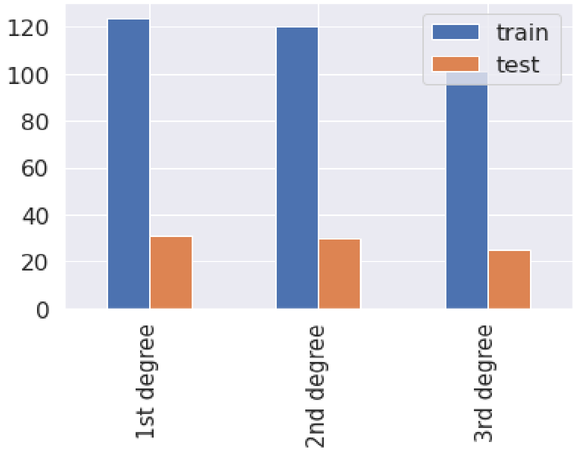

Fig.2. Original train and test sets according to three types of burn degrees

3. Description of the Method

Images are represented as arrays in our data. Colored images are a mixture of red, blue and green (RGB). The pixels of the images are tiny blocks of information arranged in the form of a 2D grid, and the depth of a pixel is the color information. Data augmentation basically shifts or transforms these pixels to get a new image.

In our data we have resized all the images to be of the shape of (150, 150, 3), where the first two numbers show the size of rows and columns of the matrix that form the 2D grid, and the third number indicating the RGB coloring. If we separate the images into three (150, 150, 1)-shaped matrices, we will get three separate red, blue and green colored pictures (Fig.3.)

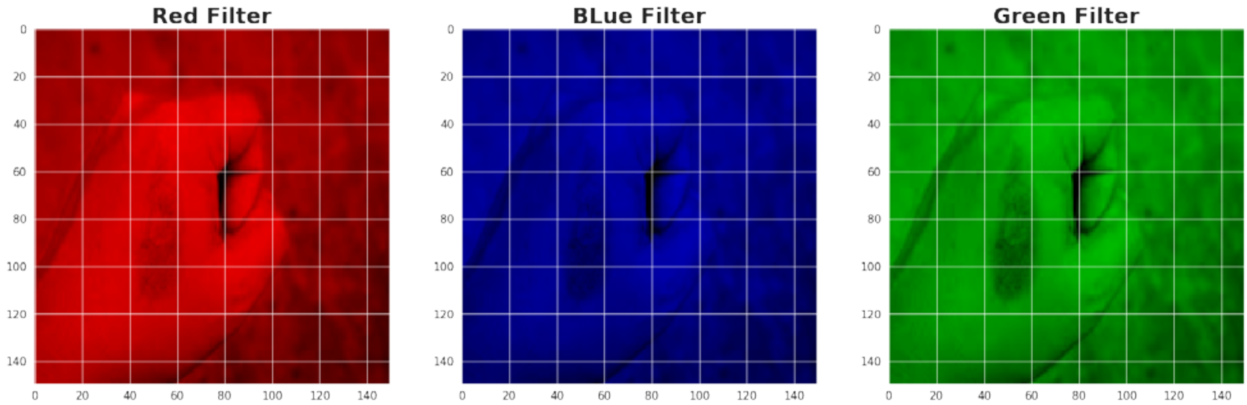

Fig.3. RGB representation of a picture.

Here are five basic and powerful augmentation techniques that we used to increase our dataset [5].

Table 1: 5 Data Augmentation Techniques Used in the program, their application and results

Data Augmentation Technique	Methods used in the program	Result of the Technique
Flip	np.fliplr(image) np.flipud(image)	Flipping images horizontally and vertically
Rotate	rotate(image, angle = 45) rotate(image, angle = -45)	Rotating images by a random angle chosen by the program
Crop	Function defined by the user to crop the main part of the image.	Randomly sampling a section from the original image, then resizing the original image size. This process is known as random cropping
Brightness adjustment	adjust_gamma(image, gamma = 0.5, gain = 1) adjust_gamma(image, gamma = 2, gain = 1)	Changing the original image's darkness/brightness
Noise	random_noise(image)	Adding noise to the image, hence having blurred image with slightly different pixel values

The flip and rotate transformations are alike. Particularly, flipping the image horizontally or vertically is the same thing as rotating the image by 90 or 180 degrees. In order to rotate our images around its center by specified number of degrees, we take the RGB values at every 2D location, rotate it as needed, and then write these values in the new location. Thus, having the location coordinates x and y, we apply the transformation matrix and get the new location for the same RGB value. The calculation is done with multiplication of the old coordinates with the transformation matrix, as shown in formula 1.

$$\begin{bmatrix} x^* \\ y * \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix},$$

where θ is the angle by which we want to rotate, x* and y* are the new coordinates, and x and y are the old ones. In our case we have used 90, 180, 45 and -45 degrees. After having the images flipped horizontally and vertically, rotation of the images by 45 and -45 degrees is the most effective, because two other lesser degrees would result in the same original image, e. g. 10 degrees, and, rotating by a large degree will be very close, as to flipping horizontally or vertically. Hence, rotating by 45 degrees, which is the mean value of 0 and 90 degrees, is the best choice for our images.

After these 2 techniques we generate four rotated images, which we append to our training set.

Cropping could be done by randomly choosing a smaller rectangle on our image and cutting out that part. However, for the crop technique the essential condition is having the burn image fully displayed on our image even after cropping it. In order to reach this goal, in our code we first divide the y axis of our image into three equal parts, having the low, middle and high parts. Then we choose a random number that would belong to the low part, and another random number from the high part. As a result, we have two points on y axis. Same goes for the x axis. By having 2 points on the x axis and 2 on the y axis, we then form the rectangle that includes the middle part of the image and which will be cropped out. By doing so we generate one image from each original one and add to the training set.

Brightness adjustment means adjustment of the RGB value. In our technique we have multiplied each of R, G, B values by two constants. In the first case by 0.5, and as 0.5 is less than 1, then it made our images darker. In the second case we multiplied by 2, which made our images brighter. As a result of this technique, we generated two more images.

The last thing we have applied to our images is adding some blur. Most common are box blur and Gaussian noise. For the box blur we took a kernel, which is a 5x5 matrix, rolled on the image, and took the average color of pixels inside that matrix. As for the Gaussian noise, we add random variables from standard normal distribution to our RGB values. Based on these two techniques we get two new images, therefore, overall nine images from each image [10][11].

With application of our data augmentation techniques on each image (Fig. 4) we generated 3105 new images and, hence, together with the original data a new training set consisting of 3450 images [6][7] was produced.

In order to be able to fit into the CNN model, each image should preserve the size of the original dataset, which is 3-dimensional array of (150, 150, 3)-shape. However, in comparison with the original dataset, not all the newly created ones have the necessary shape. It occurs as a result of transforming the arrays. CNN models should have inputs of the same size, hence, it is crucial to bring all the data into the same shape before fitting to the model. The reshaping was done individually for each image after their transformation. After changing the sizes of the images, the information isn't lost: it is just demonstrated via different shaped arrays [9].

For the transformations such as flip, crop, brightness adjustment, Gaussian noise, etc., the images will retain all the information. For some cases, such as for the rotations, the image does not have any information about things outside its boundary. As we see in Fig.4, the picture in the 2nd row and 1st column has black fillings outside its boundary. In such cases we need to make some assumptions. There are different ways of doing so:

Constant - filling the unknown region with some constant value

Edge - the edge values are extended after the boundary

Reflect - the image pixel values are reflected along the image boundary

Symmetric - at the boundary of reflection, a copy of the edge pixels is made

Wrap - the image is repeated beyond its boundary.

As our images are mainly taken in a monochromatic background, the space beyond the image's boundary is assumed to be the constant 0 at every point and is displayed with black color [10].

After the application of all the above-mentioned techniques and bringing all the images to the same size, for each image we get nine copies with a slight difference. The augmented images are demonstrated in Fig.4.

There are other types of data augmentation such as Generative Adversarial Networks (GAN). It is questionable why NN data augmentation is not preferred while using CNN model. The answer lies in the data itself. The images that we have are not easily distinguishable even by the human eye. For example, if we were to classify buildings from forests, then with GAN we could generate the same picture of the building in summer, autumn, spring, winter and, hence, have four more pictures in this case. In our case by generating new images using GAN, we can turn an image with 2nd degree burn into a 3rd degree burn image and decrease the accuracy of the CNN model.

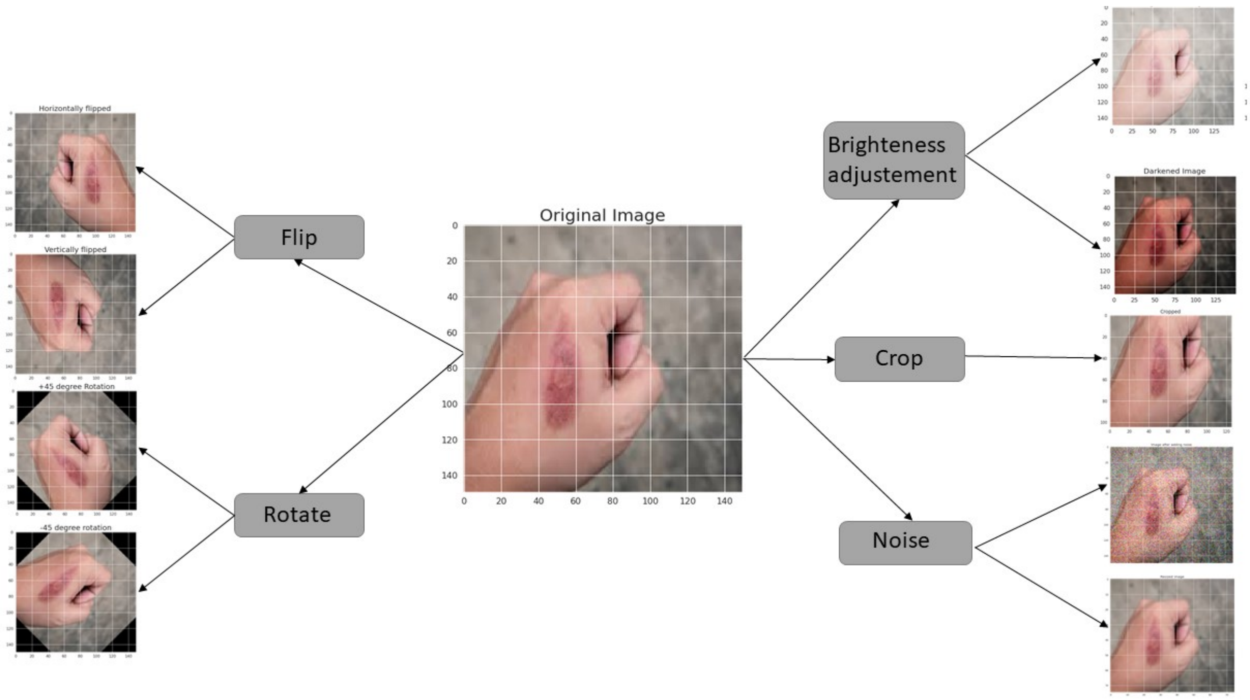

Fig. 4. Application of the Data augmentation techniques on an image.

After the transformations we have two training datasets. The first one is the original one containing 345 unique images, and the second one is the augmented, containing overall 3450 images. In order to understand whether data augmentation is beneficial for the application of the CNN model, we need to plug the original training set of 345 to the model, get the accuracy and then plug in the second augmented training set of 3450 and get the accuracy. The result will be seen by comparing the two accuracies. In our case we applied convolutional neural networks and got 59% accuracy out of original dataset and 75% accuracy out of the augmented dataset.

4. Conclusion

Data augmentation gives the opportunity to add more training data into the model, prevent data scarcity for better models, reduce data overfitting, create variability in data and resolve class imbalance issues in classification. For classification of human skin burns, the newly generated images should not lose much information from the original image. In comparison with our techniques, the other data augmentation tool - generative adversarial network, generates synthetic copies of an image, basically mixtures of images. In our case it is not expedient, as the difference between classes is very slight, and even a small mixture of images will result in bad input data.

Besides all these advantages the transformation of the images reduces costs of collecting and labeling data.

To fully take advantage of the data augmentation, the below mentioned steps were taken:

- Observe data to understand which augmentation tools are better to use;
- Implement and apply those tools on each image to produce 3105 new items (from 345 original images) adding to our training set;

- Create two training datasets. The first one is the original dataset consisting of 345 images and the second is the augmented dataset, containing 3450 images (from which 345 were the original images and 3105 newly generated);
- Fit both training sets into a CNN and get the corresponding models;
- Test the two models on the test set and get the accuracies for each.

Thus, as a result of data augmentation the training set increased from 345 items to 3450 and the CNN accuracy from 59% to 75%.

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Մեքենայական ուսուցմամբ տվյալների բազայի ընդլայնման մեթոդի մշակումը և կիրառումը

Արեն Կ. Մայիլյան

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Ամփոփում

Նեյրոնային ցանցի մոդելների էֆֆեկտիվությունը պայմանավորված է տրված պարամետրերով և մուտքային տվյալներով: Արտաքին գործոններով պայմանավորված դժվարություններից և սահմափակումներից ելնելով՝ նեյրոնային ցանցի մոդելների համար հավաքագրված տվյալները սովորաբար բավարար չեն հատկապես պատկերների դեպքում: Այս խոչընդոտը հաղթահարելու նպատակով օգտագործվում են մեքենայական ուսուցման մեխանիզմներ, որոնք թույլ են տալիս ընդլայնելու տվյալների նախնական բազան: Վերջինս ներկայացնող մուտքային պատկերի վերափոխումամբ ստեղծվում է պատկերների բազմազան հավաքածու, հնարավորություն տալով արդյունավետորեն լուծելու դասակարգման, նույնականացման և սեգմենտավորման/մասնատման խնդիրներ:

Բանալի բառեր՝ Մեքենայական ուսուցում, կոնվոլյուցիոն նեյրոնային ցանցեր, տվյալների ընդլայնում, այրվածքի աստիճաններ:

Разработка и применение метода расширения базы данных с помощью машинного обучения

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Аннотация

Эффективность моделей нейронных сетей (NN) зависит от заданных параметров и входных данных. Из-за сложности и ограничений, обусловленных внешними факторами, данных для моделей NN, довольно часто бывает недостаточно, особенно в случае изображений. Чтобы решить эту проблему, обычно используется расширение данных для увеличения их набора. Преобразование входных данных создает разнообразный набор изображений из небольшого набора изображений для классификации, обнаружения или сегментации изображения.

Ключевые слова: Машинное обучение, сверточная нейронная сеть, нейронные сети, расширение данных, степени ожога.

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Education Through Wikipedia

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Abstract

Wikipedia belongs to education in various ways. One gains knowledge by reading Wikipedia, the other obtains profound knowledge by contributing to Wikipedia. It is the reason why educators in many countries include Wikipedia editing into their curriculum. The article is dedicated to the ecosystem of education through Wikipedia and other Wiki projects which were created by the author, developed by Wikimedia Armenia and settled in Armenia. For seven years Wikimedia Armenia has been implementing Wikipedia Educational projects in different rural regions ofArmenia and hopefully will continue its development. The system offers permanentcreative learning for teachers, as well as deep and interdisciplinary education on their future field of engagement with students. It revolutionarily changes the attitude of teachers and students towards education. It facilitates the teacher and student relationships. It also changes students' interrelation from contest to cooperation. It shifts the attention of educational players from marks to topics' perception. Of course, the most valuable advantage of this approach is that teachers improve their knowledge continuously and students, even not the smart ones gain comprehensive knowledge. This ecosystem is constantly improved based on statistical surveys. The components of the ecosystem were honored as the coolest Wikimedia projects and registered as trademarks: Wikicamp, Wikiclub. In the current article the full overview of education through wiki projects is given. The detailed description and innovative solutions on the challenges of today's education will be introduced in the upcoming articles of the publication issue.

Keywords: Wikipedia, education, Wikimedia projects, flipped classroom, teachers' training, non-formal education.

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1. Education is in Crisis around the World

How many people have you met who have graduated from school but are illiterate or at best have superficial knowledge despite having excellent marks at school? How many have graduated from universities but cannot even formulate their thoughts? With the new technologies, it becomes a total disaster!

You can get any information from the Internet. You don't really need to think or explore thoroughly.On the other hand, the technological era will automate many human skills. So, in thefuture, the most desired and required people will become interdisciplinary professionals, while subjects and concepts in schools are still studied as entirely separate things. Textbooks, even innovative ones, shortly become inappropriate.

Teachers - being used to repeat the same content every day, are not able to handle scientific and technological innovations. With the COVID-19 pandemic, it turns even worse.

To fill the gaps in the knowledge of teachers, many webinars and trainings are being organized by educational institutions, but little is improved. Generally, after the training course, teachers mostly continue using the traditional methods of teaching. However, learning through Wikipedia is entirely different.

In many countries, Wikipedia is used for educational purposes differently. In the beginning, everyone faced the same problem that Wikipedia is not trustworthy - it includes misinformation, which is why it should not be used either by students or by teachers [1, 2]. However, the picture gradually changed internationally as many teachers and students began using Wikipedia in their educational processes. Some began promoting specific language content on Wikipedia concentrating on scientific topics [3]. Others suggested allowing students to use Wikipedia, as a source for information by teaching them the methods of discerning the truth from misinformation using third sources [2]. As a result, Wikipedia was gradually incorporated into the educational processes of universities in different countries, for example, the USA and Canada. Wiki Education, which supports the Wikipedia Education Process runs a program in which university instructors assign articles to their students on specific topics to add on Wikipedia [4]. Professors from Texas A&M University-Kingsville consider that wiki assignments help the students to gain a deep understanding of certain content and to gain knowledge collaboratively [5]. A high school teacher from Pennsylvania considers Wikipedia as a tool to conduct research by using its sources, bibliography and external sources [6].

The approaches of using Wikipedia in educational processes are quite different. Based on international experience, Wikimedia Armenia localized the practice of teaching with Wikipediaby developing new approaches and methods of educating through Wikipedia - mainly incorporating Wikipedia in educational processes in a way that substantially benefits both the students and teachers and Wikipedia itself.

2. Wikipedia Belongs to Education

Below are the statements on why reading or contributing to Wikipedia is the best way to learn and digest the topic of one’s study.

1. Wikipedia is contributed from all over the world. Thus, you get the information that is created, updated and enhanced by thousands of humans.
2. Topics are not connected to a singular subject (physics, chemistry or biology, etc.), it describes an object or a concept.
3. All subjects or concepts highlighted in the content are linked to corresponding articles on Wikipedia, which help people understand each word’s meaning, therefore, understanding the content profoundly, too.

4. During creating or developing Wikipedia articles, one studies the subject deeply and learns continuously to describe thoughts even better.
5. If one translates articles constantly, then he/she enhances the vocabulary of the foreign language and the translation skills becoming fluent in both languages.
6. While participating in discussions one becomes more patient enhancing the listening skills and taking into account the others' opinions.
7. Contributions that one does are visible to everyone, so the superiors can see the level of one's knowledge and contributions entirely.
8. Education managers and authorities have an opportunity to uncover and evaluate teachers' potential through statistics and contributions on Wikipedia.
9. In a few years, one can achieve the top professional level in the world in a specific field.

3. Ecosystem of Education through Wikipedia

Considering the international experience, Wikimedia Armenia has developed an ecosystem of Education through Wikipedia, which was almost seven years being run in different urban and rural regions of Armenia. The system provides permanent creative learning for teachers and an all-embracing interdisciplinary education for students in their future field of engagement [7].

Fig. 1 Education through Wikipedia.

The ecosystem developed by the author and implemented by Wikimedia Armenia consists of several components which enable to fully incorporate Wikipedia into educational processes of schools and universities and to enrich Wikipedia itself. These components are teachers' training, Wikiclubs, Wikicamps, Wikiclassrooms, and university collaborations. The novelty of the ecosystem is that it addresses the "Education with Wikipedia" question via several target groups in terms of age and profession. Talking about teaching Wikipedia to teenagers, Samir Elsharbaty, then Wikimedia Foundation's Communications Intern, announced that it is challenging to train teenagers who are not acquainted with research which is vital for Wikipedia [8]. While most of the editors worldwide are adults, Wikimedia Armenia has adopted another approach - alongside others, target teenagers to develop their researching abilities and language skills from young age by using and working on Wikipedia.

Teachers’ training is the most important part of education via Wikipedia [9]. We train hundreds of teachers to contribute to Wikipedia encouraging them to bring these skills to the classroom and to develop their own way of working with students via Wiki projects.

The Ministry of Education in several countries includes Wikipedia training as a component of the accreditation process. In some countries, the Ministry of Education is workingwith the Wikimedia Foundation’s educational team or with Wikipedia Educational Foundation.

After Wikipedia training courses teachers can run a Wikiclub or a Wikiclassroom. Wikiclubs are an alternative and non-formal educational activity. The idea to open Wikiclubs in different regions of Armenia was born out of the concern to provide youngsters with meaningful activity and make education a habit.

More than seven years of experience running Wikiclubs shows that it corresponds to the needs of all students: smart or not. Each student can find his or her niche and grow! They get profound knowledge, they acquire learning skills such as literacy, concentration, perception skills and critical thinking abilities.

In Wikiclubs we use Wiktionary - a comprehensive dictionary - for improving children's literacy, we use Wikisource - an online digital library - for attention and literacy. Wikipedia andWikiversity are great tools for improving concentration, formulating thoughts, and deepening perception, diligence and thinking abilities.

Wikiclubs' approach has been copied by many countries, for example the GLAM Macedonia User Group effectively learned and localized the idea of Wikiclubs [10]. There are Wikiclubs in schools, universities, and GLAM institutions.

Wikicamps are the most inspiring and encouraging part for students. WikiCamp: a short vacation for students where they learn about Wikimedia projects and edit Wikipedia and also play sports, go hiking, play intellectual games, and do other amusing activities. To get Wikicamp tickets students contribute to Wikipedia during the year.

Wikiclub-Wikicamp combination enhances students' interest in learning. While in schools the learning is estimated via marks, in each project of the ecosystem students receive points depending on the quantity and quality of their contribution to Wikipedia, Wiktionary, and Wikisource. The statistics are provided on a regular basis - each month. Instead of marks students get prizes appropriate for their input. The highest prize is a ticket to Wikicamp. To attend Wikicamp students have to work during the whole academic year. Therefore, sooner or later learning is becoming a habit.

Wikiclassroom is a more evaluated level of education through Wikipedia. It needs the joint effort of teachers of various subjects: native and foreign languages and interconnected subjects related to a certain topic. Students are working on the article jointly in a group along with teachers. Flipped classroom method is used during lessons. First attempt to implement this idea was during a biology class in Lernapat school of Lori province. The school teachers were so excited and inspired that they moved their course to the Wikiclassroom model [11].

University collaboration is common around the world, as in the cases of the USA and Canada where within the framework of Wikipedia Student Program Wiki trainers provide assistance to university professors to run the course through Wikipedia [4]. In Armenia, it is more successful in linguistic and medical universities. Brusov State University has been running internship and diploma programs through Wikipedia for almost three years [12]. It significantly enhances students' vocabulary and translation skills.

4. Conclusion

It is time to bring this opportunity to schools and not via the utilities but via the creativityof each teacher and individual educators.

To influence education significantly, we need to use the top-down approach. We need permission from the authorities to reach each school, each university, each teacher, each lecturer and each student, therefore each human being.

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Կրթություն Վիքիպեդիայի միջոցով

Սուսաննա Մ. Մկրտչյան

ՀՀ ԳԱԱ Ինֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտ
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Ամփոփում

Վիքիպեդիան և կրթությունը փոխկապակցված են․ կարելի է գիտելիք ստանալ Վիքիպեդիա ընթերցելով և խորը գիտելիք ձեռք բերել՝ Վիքիպեդիայում ներդրում կատարելով: Այդ իսկ պատճառով բազմաթիվ երկրների մանկավարժներ ներառում են Վիքիպեդիան իրենց ուսումնական պլանում: Այս հոդվածը ներկայացնում է Վիքիպեդիայի և այլ վիքի նախագծերի միջոցով կրթական էկոհամակարգ, որը ստեղծվել է հեղինակի և զարգացվել Վիքիմեդիա Հայաստանի կողմից: «Վիքիմեդիա Հայաստանն» իրականացրել է Վիքիպեդիայի կրթական նախագծեր Հայաստանի տարբեր գյուղական շրջաններում: Համակարգը հնարավորություն է ստեղծում ուսուցիչների համար՝ շարունակ ստեղծագործ կրթվելու, իսկ աշակերտների պարագայում՝ խորացնելու իրենց միջառարկայական գիտելիքները ապագա մասնագիտական ոլորտներում: Այն արմատապես փոխում է ինչպես ուսուցիչների և աշակերտների վերաբերմունքը կրթության նկատմամբ, այնպես էլ ուսուցիչ-աշակերտ փոխհարաբերությունը: Այս մոտեցման շնորհիվ աշակերտների միջև մրցակցությունը փոխակերպվում է համագործակցության: Այն շեղում է սովորողների ուշադրությունը գնահատականներից դեպի թեմայի ընկալում: Հիրավի, այս մոտեցման ամենամեծ առավելությունն այն է, որ ուսուցիչները շարունակաբար բարելավում են իրենց գիտելիքները, իսկ աշակերտները ձեռք են բերում խորը գիտելիքներ: Էկոհամակարգի բաղադրիչներից Վիքիճամբարը և Վիքիակումբը ճանաչվել են Վիքիմեդիա լավագույն նախագծեր: Այստեղ ներկայացված է Վիքիմեդիա նախագծերի միջոցով կրթվելու ամբողջական պատկերը, մերօրյա կրթության մարտահրավերների առավել մանրամասն նկարագրությունն ու նորարարական լուծումները կներկայացվեն առաջիկա հոդվածներում:

Բանալի բառեր՝ Կրթություն, Վիքիպեդիա, Վիքիմեդիա նախագծեր, շրջված դասարան, ուսուցիչների դասընթաց, ոչ ֆորմալ կրթություն:

Экосистема образования через Википедию

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Аннотация

Википедия во многом способствует образованию. Один получает знания, читая Википедию, другой - имея знания, внедряет свой вклад в Википедию. Это и есть главная причина, по которой преподаватели во многих странах включают редактирование Википедии в свои учебные программы. Статья посвящена инновационной экосистеме образования созданной автором, которая использует Википедию и другие вики-проекты как платформу для креативного обучения. В течение семи лет «Викимедиа Армения» реализует образовательные проекты Википедии в различных сельских регионах Армении и продолжаетт развивать.Система предлагает творческое обучение для учителей и междисциплинарное образование для учащихся в области их будущих профессий. Это меняет отношение учителей и учеников к образованию и, соответственно, меняются их взаимоотношения..Такой подход позволяет ученикам перейти от соперничества к сотрудничеству и переключить внимание на восприятие тем. Компоненты экосистемы Викилагерьи Викиклуб. были отмечены как самые крутые проекты Викимедиа и зарегистрированы как торговые марки. В статье дается укрупненная картина обучения через вики-проекты.

Ключевые слова: Википедия, образование, проекты Викимедиа, перевернутый класс, подготовка учителей, неформал.

Կանոններ հեղինակների համար

ՀՀ ԳԱԱ ԻԱՊԻ “Կոմպյուտերային գիտության մաթեմատիկական խնդիրներ” պարբերականը տպագրվում է 1963 թվականից: Պարբերականում հրատարակվում են նշված ոլորտին առնչվող գիտական հոդվածներ, որոնք պարունակում են նոր` չհրատարակված արդյունքներ:

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