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Enhancing Symbolic Manipulation through Pairing Primitive Recursive String Functions and Interplay with Generalized Pairing PRSF

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Abstract

In earlier studies, the notion of generalized primitive recursive string functions has been presented, and their connections with abstract paring-based primitive recursive string functions have been investigated. Our study is centered around establishing a fundamental theorem that states a connection between these two distinct sorts of functions. The theorem specifically establishes that the universal definition of every generalized pairing primitive recursive string function is contingent upon its correspondence with a conventional(abstract) pairing primitive recursive string function. This article introduces innovative concept of Pairing Primitive Recursive String Functions (P-PRSF) for manipulating and interacting with word pairs. Based on the principles of primitive recursion and pairing functions, P-PRSF enables the extraction, transformation, and combination of word components. The proposed theorems validate the effectiveness of P-PRSF in capturing relationships within word pairs. Moreover, the interplay between P-PRSF and Generalized Pairing PRSF (GP-PRSF) extends the concept to involve more intricate interactions.
Keywords: Word pairing, PRSF, Generalized Pairing PRSF, Superposition, Alphabetic PRSF.
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1. Introduction

The foundational principles of basic recursive string functions have provided a framework for comprehending the computing capacities of functions that manipulate individual words from a specified alphabet[1, 2, 3]. This novel idea introduces the notion of generalized pairing primitive recursive string functions (GP-PRSF) and endeavors to unveil the intricate connections between these advanced functions and their conventional counterparts, known as pairing primitive recursive string functions (P-PRSF) in the existing literature[4, 5].

The motivation behind this investigation stems from an inherent curiosity: what occurs when we surpass the limitations of individual words and engage in the process of manipulating pairs of words? In the context of this endeavor, the notion of GP-PRSF arises, enabling the integration of indeterminate functions inside this novel framework. This study aims to explore the fundamental connections between GP-PRSF and P-PRSF through an examination of a crucial theorem that provides insights into the circumstances under which these functions are uniformly defined.

2. Pairing Primitive Recursive String Function (P-PRSF)

The concept of Pairing Primitive Recursive String Functions (P-PRSF) emerges as a formalism to operate on pairs of words from a given alphabet. P-PRSFs build upon the foundation of traditional Primitive Recursive String Functions (PRSF) [5, 6] and extend their capabilities to address the complexities of word pairing.

Let $A = \{a_1, a_2, .a_p\}$ be a set of alphabets comprising $p > 1$ distinct symbols. The PRSF function F is defined as operating on pairs of words, (P, Q), where P and Q are words $\in A$. The aim of F is to produce an output based on the input pair. Further, introducing $\Pi_1(P, Q) = P$ and $\Pi_2(P, Q) = Q$ are basic pairing functions that extract the first and second components of the pair, respectively[5].

An undefined basic pairing function: In the context of the extension for word pairing as generalized word pairing (GPRPF), an undefined word pair function through $U(P, Q)$ returns an undefined value for any input pair (P, Q). It may refer to a scenario where a function is not defined for certain input pairs as similar to an undefined value for PRSF [7]. This could be due to possible specific conditions or restrictions on the input pairs and also serve as a foundation for generalized pairing.

2.1 Operators for Pairing Functions

Superposition: If F is a pairing function, and $G_1, G_2, , G_n$ are pairing functions, then the superposition F^* of F with $G_1, G_2, , G_n$ is defined[5] as Equation 1 below:

$$F^*(P, Q) = F(G_1(P, Q), G_2(P, Q), , G_n(P, Q)). \tag{1}$$

In the case of only two pairs of words $(P1, P2)$, it can be used as $F^*(P1, P2) = F(G_1(P1, P2), G_2(P1, P2))$, where only two representations are paired in terms of G_1 and G_2.

Alphabetic Primitive Recursion for Pairing Functions: If F is a pairing function, and $H_1, H_2, , H_p$ are $(n + 2)$ dimensional pairing functions, then the primitive recursive pairing function[5] F^+ of F with $H_1, H_2, , H_p$ is defined "for some" $1 \leq i \leq p$ as Equations 2 and 3:

$$F(P, Q) \qquad \text{if } R = \Lambda, \tag{2}$$

$$H_i(P, Q, R, F^+(P, Q, R)) \qquad \text{if } R = Qa_i \text{ for some } a \in A. \tag{3}$$

To better understand how operators use word pairing through P-PRSF, we perform these operations as according to Table 1, where we'll perform operations over three different input pairs. The output for each input pair can be understood as a word that has been constructed through a process that involves paired words and the application of functions. In the context

Table 1: Perform P-PRSF through superposition and Alphabetic PRPF over different input pairs.

Operations	**Definitions**	**Input1**	**Input2**	**Input3**
		$\langle ab, ba\rangle$	$\langle a, bb\rangle$	$\langle ba, aba\rangle$
Π_1		ab	a	ba
Π_2		ba	bb	aba
Superposition	with the following assumptions as $\begin{cases} G_1(P,Q) = \Pi_2 \\ G_2(P,Q) = S_g(Q) \\ \text{to extract the first symbol from the 2nd component of word pairs} \\ F(P,Q) = \Pi_1 \end{cases}$ $F^*(P,Q) : F(G_1(P,Q), G_2(P,Q))$	ba	bb	aba
Alphabetic PRPF	$F^+ = F(P,Q)$ as Equation 2 for $R = \Lambda$	ab	a	ba
Alphabetic PRPF	If $R = Qa_i$ as assumed **ab** $\begin{cases} H(P,Q,R,F^+(P,Q,R)), \\ F^+(P,Q,R) \text{ can be evaluated} \\ \text{recursively as} \\ \Pi_3(P1, P2, Qai) = \Pi_2(P1, P2)[7] \end{cases}$	H(ab, ba, **ab**, ba)	H(a, bb, **ab**, bb)	H(ba, aba, **ab**, aba)
Alphabetic PRPF	For simplicity(*), we assumed the H function to extract a word with repeated occurrences from input word pairs	ba	bb	aba

of word pairing and incorporating some assumptions in terms of G and H, this output simply represents an element that emerges from the relationships between the original input words, with the recursive process playing a role in shaping these outcomes.

Theorem 1. (P-PRSF for Word Pairing) *For any word pairing (P,Q) and given pairing functions F and G as defined, the Alphabetic Primitive Recursive String Function* F^+ *effectively captures and manipulates the relationships within word pairs.*

Proof. The previously proposed lemma [5] along with the proposed Theorem [5] was the foundation for the equivalence between GPRSF and PRSF. The connection lies in the concept of "S-image" or the superposition operation[4],[5],which transforms a string function into a new one. This concept is analogous to the idea of combining and manipulating word pairs using the proposed P-PRSF operations. Therefore, this theorem can be proved by induction on the length of the third component R of the input (P, Q, R).
Base Case: For $R = \Lambda$, the base case of Alphabetic Primitive Recursion applies $F^+(P, Q, \Lambda) = F(P, Q)$. This effectively extracts and manipulates the components of the input word pair according to F, demonstrating the foundational operation for word pairing.
Inductive Hypothesis: Assume that for any non-empty word $R = Qa$ of length n, $F^+(P, Q, Qa)$ effectively captures and manipulates the relationships within the word pair

(P,Q), guided by the pairing functions F and G. Consider the word $R = Qab$ of length $n+1$. By the inductive hypothesis, $F^+(P,Q,Qab)$ is constructed through interactions between the word pair (P,Q), R, and the results of previous recursion steps.
Steps: i. Apply $H(P,Q,Qab,F^+(P,Q,Qab))$: This step involves interactions between P, Q, Qab, and $F^+(P,Q,Qab)$, effectively capturing relationships within the pair and guiding the construction of the result. ii. The recursive process navigates through the components of R and their interactions with F^+, eventually yielding a word that reflects the relationships between the original word pair.
By induction, for any non-empty word $R, F^+(P,Q,R)$ effectively captures and manipulates the relationships within the word pair (P,Q). The theorem is proven through induction, demonstrating that Alphabetic PRSF F^+ effectively captures and manipulate relationships within word pairs. ■

Theorem 2. (Preservation of Word Pairing Relations) *For any word pairing (P,Q) and given pairing functions F and G as defined, the Alphabetic PRSF F^+ preserves the inherent relationships and interactions within the word pair.*

Proof. $F^+(P,Q,R)$ is demonstrated to accurately preserve and reflect the relationships between the components of the input word pair (P,Q) and the recursive components R. Through Base Case: For $R = \Lambda$, $F^+(P,Q,\Lambda)$ evaluates to $F(P,Q)$, capturing the initial relationship between the components of (P,Q). Using Recursive Case: For $R = Qa$, where a is a symbol from the alphabet A, $F^+(P,Q,Qa)$ involves interactions between P,Q,Qa and the result of F^+ for the previous recursive component. This step accurately reflects the inherent interaction between the components of the word pair and guides the outcome based on the chosen pairing functions F and G.
Considering both the base and recursive cases, it becomes evident that $F^+(P,Q,R)$ effectively preserves the relationships and interactions within the word pair (P,Q) and the recursive components of R. The preservation of the word pairing relations is established through the accurate reflection of interactions and relationships within the word pair as guided by Alphabetic Primitive Recursive String Function F^+ . This mathematical proof reinforces the proposed idea of Pairing Primitive Recursive String Functions for word pairing scenarios. ■

Lemma 1. (Preservation of S-Image under P-PRSF Operations) *For any word pairing (P,Q) and given pairing functions F and G as defined, the application of Alphabetic PRSF F^+ to a pair of string functions F and G preserves the **S-image** property.*

Proof. Consider a string function F and its S-image F^*. Applying the Alphabetic PRSF F^+ on F and G to form $F^+(F,G)$, the new function $F^+(F,G)^*$ is obtained. As in Base Case: by applying F^+ to F and G retains the S-image property for $F^+(F,G)$, as the base case and operations of F^+ are defined consistently with the S-image property. Continue with Recursive Case: that preserves the S-image property under F^+, as it relies on the same underlying pairing functions F and G that maintain the S-image property.
By induction, the lemma demonstrates that the application of Alphabetic PRSF F^+ to the pairing functions F and G retains the S-image property, analogous to the preservation of S-image under generalized primitive recursive string functions. While the previous lemma (2015) deals with generalized primitive recursive string functions and their S-images, the

proposed new lemma focuses on the preservation of S-image under the proposed P-PRSF operations for word pairing. ■

Lemma 2. (Composition of P-PRSF is P-PRSF) *For any Pairing Primitive Recursive String Functions (P-PRSF) F and G, the composition $F \circ G$ is also a P-PRSF.*

Proof. The composition of P-PRSF functions F and G retains the properties of P-PRSF, namely the basic functions, superposition, and alphabetic primitive recursion. Both F and G are P-PRSF, they inherently preserve the basic functions (pairing functions Π_1 and Π_2) as well as alphabetic primitive recursion and superposition operations. Therefore, their composition $F \circ G$ also preserves the basic functions. The superposition operation is defined as $F^*(G^*(P, Q))$, where F^* and G^* are S-images of F and G, respectively. Both F^* and G^* are primitive recursive string functions in the usual sense due to the properties of P-PRSF. Since the composition of two primitive recursive functions is itself primitive recursive, $F \circ G$ retains the property of superposition. In respect of Alphabetic PRSF, F and G can also be defined as with respect to pairing functions Π_1 and Π_2. The composition $F \circ G$ is defined as $F^+(G^+(P, Q, R))$, where F^+ and G^+ are Alphabetic PRSF. The composition retains the property of alphabetic primitive recursion as it operates on the components and interactions of the word pair based on the P-PRSF operations.
By establishing the properties of basic functions, superposition, and alphabetic primitive recursion for the composition $F \circ G$, we conclude that the composition of P-PRSF functions is also a P-PRSF. This lemma demonstrates that the composition of P-PRSF functions adheres to the same principles and operations as individual P-PRSF functions. This supports the idea that the proposed operations for word pairing maintain their validity even when combined in a composite manner. ■

3. Interplay with Generalized Pairing PRSF (GP-PRSF)

Now extend the notion of Pairing Primitive Recursive String Functions (P-PRSF) to involve interactions with generalized versions of these functions[4],[6],[5]. This would allow us to combine the foundational idea of word pairing with more complex interactions based on GP-PRSF.
Let's denote the Generalized Pairing PRSF as $H(P, Q)$, where H is a function that involves interactions between word pairs (P, Q) and is guided by specific rules and operations. The interactions can be more intricate than basic pairing, incorporating additional considerations or conditions. Here, H is a function that takes two words P and Q as input and performs complex interactions between them according to H. The proposed concept involves applying Alphabetic PRSF F^+ to a pair of P-PRSF F and GP-PRSF H. This results in the formation of a new function $F^+(F, H)$, which captures the interplay between the simpler word pairing operations and the more intricate interactions guided by H. The focus is on demonstrating that the operations of $F^+(F, H)$ retain the desired properties of both P-PRSF and GP-PRSF, thus effectively combining the foundational and advanced aspects.

Example 1 *Now we'll define a GP-PRSF that measures the similarity between two words based on their character composition. It is actually a very well-developed idea where NLP researchers achieved effective results in this task, but our focus is only on the interplay between P-PRSF and GP-PRSF. We'll then apply the interplay concept to combine this*

GP-PRSF with a P-PRSF. Let's define a GP-PRSF $S(P, Q)$ that measures the similarity between two words P and Q based on their character composition. The function S calculates the number of common characters between P and Q, normalized by the length of the longer word. The formula for S can be defined[2] as the following equation:

$$S(P, Q) = \frac{\textit{Number of common characters}}{max(\textit{length of } P, \textit{length of } Q)}.$$

Given a P-PRSF $F(P, Q)$ that extracts the first word of a pair ($F(P, Q) = P$), the interplay function $F^+(F, S)$ would involve applying $F(P, Q)$ to the first word of the pair and then calculating the similarity S between that word and the second word Q. Suppose we have the following word pair: P="apple" and Q="ample". Using The GP-PRSF S calculates the similarity between P and Q as 3/5=0.6, now to Interplay with GP-PRSF $F^+(F, S)$ P-PRSF $F(P, Q) = P$ to P, which results in P="apple". Then, we calculate the similarity S between P and Q as $F^+(F, S)(P, Q)$=$S(P, Q)$=0.6.
As above, The interplay between the basic pairing operation and the GP-PRSF involves extracting the first word P, calculating the similarity between P and Q, and obtaining a numeric value that represents the degree of similarity between the words.

4. Conclusion and Future Directions

The concept of Pairing Primitive Recursive String Functions (P-PRSF) within the domain of word pairing presents a systematic framework for the manipulation and interaction of word pairs. By integrating fundamental principles of basic recursion[1],[7] with the inventive methodology of pairing functions, this notion facilitates the extraction, modification, and integration of linguistic elements. The theorems and lemmas offered in this study provide evidence for the soundness of P-PRSF, demonstrating its efficacy in accurately representing and maintaining connections between word pairs. By examining the relationship between P-PRSF and the more detailed Generalized Pairing PRSF (GP-PRSF), the idea expands its practicality to encompass complex interactions. The notion of interplay provides opportunities to explore a wide range of applications, wherein fundamental word pairing procedures may be integrated with sophisticated interactions driven by particular rules. The inherent capacity of this interaction facilitates the development of novel functionalities that include the fundamental and intricate procedures.
The scope of future investigation may involve the advancement of algorithms that utilize the P-PRSF idea in order to facilitate activities such as text analysis, natural language processing, and data transformation. Potential applications of this technology include similarity calculations, text production, and pattern recognition. The study focuses on advanced interaction models that aim to investigate a range of GP-PRSF models for intricate interactions between pairs of words. This exploration has the potential to yield novel functionalities that effectively capture semantic links, contextually-aware transformations, and sentiment-based operations. The practical application of the P-PRSF idea to real-world challenges has the potential to yield innovative solutions. For instance, the use of P-PRSF can provide structured procedures[4],[5] that could be advantageous in the development of automated text editing tools, content summarizing techniques, and creative writing applications.

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Խորհրդանշական մանիպուլյացիայի բարելավում՝ զուգակցելով պարզունակ ռեկուրսիվ լարային ֆունկցիաները և փոխազդելով ընդհանրացված PRSF զուգավորման հետ

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Ամփոփում

Ավելի վաղ ուսումնասիրություններում ներկայացվել է ընդհանրացված պարզունակ ռեկուրսիվ լարային ֆունկցիաների հասկացությունը, ինչպես նաև ուսումնասիրվել են դրանց կապերը վերացական զուգավորման վրա հիմնված պարզունակ ռեկուրսիվ լարային ֆունկցիաների հետ: Մեր ուսումնասիրությունը կենտրոնացած է հիմնարար թեորեմի հաստատման վրա, որը կապ է հաստատում այս երկու տարբեր տեսակի ֆունկցիաների միջև: Թեորեմը հատուկ հաստատում է, որ յուրաքանչյուր ընդհանրացված զուգավորման պարզունակ ռեկուրսիվ լարային ֆունկցիայի համընդհանուր սահմանումը պայմանավորված է նրա համապատասխանությամբ սովորական (վերացական) զուգակցվող պարզունակ ռեկուրսիվ տողային ֆունկցիայի հետ: Այս հոդվածը ներկայացնում է պարզունակ ռեկուրսիվ լարային ֆունկցիաների (P-PRSF) զուգակցման նորարարական հայեցակարգը՝ բառազույգերի հետ մանիպուլյացիայի և փոխազդեցության համար: Հիմնվելով պարզունակ ռեկուրսիայի և զուգավորման ֆունկցիաների սկզբունքների վրա՝ P-PRSF-ը հնարավորություն է տալիս բառի բաղադրիչների քաղվածքը, փոխակերպումը և համակցումը: Առաջարկված թեորեմները հաստատում են P-PRSF-ի արդյունավետությունը բառազույգերի մեջ փոխհարաբերությունները ֆիքսելու հարցում: Ավելին, P-PRSF-ի և ընդհանրացված զուգակցման PRSF-ի (GP-PRSF) փոխազդեցությունը ընդլայնում է հայեցակարգը՝ ներառելով ավելի բարդ փոխազդեցություններ:

Բանալի բառեր՝ Բառերի զուգավորում, PRSF, ընդհանրացված PRSF զուգավորում, սուպերպոզիցիա, այբբենական PRSF:

Улучшение символьных манипуляций посредством объединения примитивных рекурсивных строковых функций и взаимодействия с обобщенным спариванием PRSF

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Аннотация

В более ранних исследованиях было представлено понятие обобщенных примитивно-рекурсивных строковых функций и исследованы их связи с абстрактными примитивно-рекурсивными строковыми функциями, основанными на спаривании. Наше исследование сосредоточено вокруг установления фундаментальной теоремы, которая устанавливает связь между этими двумя различными видами функций. Теорема конкретно устанавливает, что универсальное определение каждой обобщенной спаривающей примитивно-рекурсивной струнной функции зависит от ее соответствия обычной (абстрактной) спаривающей примитивно-рекурсивной струнной функции.

В этой статье представлена инновационная концепция объединения примитивных рекурсивных строковых функций (P-PRSF) для манипулирования парами слов и взаимодействия с ними. Основываясь на принципах примитивной рекурсии и функциях спаривания, P-PRSF позволяет извлекать, преобразовывать и комбинировать компоненты слова. Предложенные теоремы подтверждают эффективность P-PRSF при обнаружении отношений внутри пар слов. Более того, взаимодействие между P-PRSF и Generalized Pairing PRSF (GP-PRSF) расширяет концепцию и включает более сложные взаимодействия.

Ключевые слова: Пары слов, PRSF, обобщенные пары PRSF, обобщенные пары PRSF, суперпозиция, алфавитные PRSF.