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Distributivity in Symmetric Constructive Full Lambek Calculus

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In [4] we have developed an algebraic semantics for symmetric constructive logic of Professor Igor D. Zaslavsky [9] devoid of structural rules and have shown how it is related to cyclic involutive FL–algebras and Nelson $\mathrm{FL}_{\mathrm{ew}}$–algebras. Because of this analogy we called the obtained calculus *symmetric constructive full Lambek calculus* (**SymConFL**) and its algebraic models *symmetric constructive FL–algebras*.

We proved that the class of cyclic involutive FL–algebras (CyInFL) is mutually interpretable with the class of symmetric constructive FL–algebras ($\mathsf{SymConFL}$). Moreover, considering **SymConFL** with the basic structural rules *exchange* (e), *weakening* (w) and *contraction* (c), we have developed analogous semantics for all variants of $\mathbf{SymConFL_S}$, where S is any subset of $\{e, w, c\}$. In particular, since $\mathbf{SymConFL_{ewc}}$ is exactly symmetric constructive logic, the class $\mathsf{SymConFL_{ewc}}$ is its algebraic semantics.[1]

Likewise, we verified that the mutual interpretability holds between the commutative subclasses $\mathsf{CyInFL_e}$ and $\mathsf{SymConFL_e}$, and the integral subclasses $\mathsf{CyInFL_w}$ and $\mathsf{SymConFL_w}$. For the contractive subclasses $\mathsf{CyInFL_c}$ and $\mathsf{SymConFL_c}$ a similar correspondence does not hold. Nevertheless, we proved the term equivalence between the class $\mathsf{SymConFL_{ewc}}$ (which we also called the class of Zaslavsky $\mathrm{FL}_{\mathrm{ewc}}$–algebras) and the class of Nelson $\mathrm{FL}_{\mathrm{ew}}$–algebras. According to the result of M. Spinks and R. Veroff [7, 8], who have introduced the variety of Nelson $\mathrm{FL}_{\mathrm{ew}}$–algebras as the termwise equivalent definition of Nelson algebras [6], Zaslavsky $\mathrm{FL}_{\mathrm{ewc}}$–algebras are term equivalent to Nelson algebras as well.

In this talk we additionally consider variants of **SymConFL** that allows one to prove the law of distributivity of conjunction over disjunction. In symmetric constructive logic ($\mathbf{SymConFL_{ewc}}$) this law is provable, but it is beyond the range of that system without weakening or contraction. We use the method independently elaborated by J.M. Dunn [1] and G. Mints [5], that consists in allowing an antecedent of a sequent to be a structure built from two kinds of structures inductively.

We have developed such systems for cyclic involutive distributive FL–algebras ($\mathsf{CyInDFL}$) and their commutative and integral variants [3]. Using the definition of symmetric constructive FL–algebras and extending it with the law of distributivity we can also expand the completeness theorem to systems with contraction.

[1] We use the naming convention adopted for variants of full Lambek calculus and their algebraic models, where subscripts stand for structural rules determining properties of fusion [2].

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