

# Discretization of Geometric Models

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## Abstract

We are interested in 2D cutting and packing problem with irregularly shaped objects. In previous work the author has presented the description of the two - dimensional modeling program of irregularly shaped objects. The next step of achieving the optimal arrangement of any irregular shape details on the surface is the discretization of geometric models. In this paper a solution to this problem is given. The solution is described in steps.

**Keywords:** Optimal cutting of materials, Two - dimensional modeling, Packing problem

## 1. Introduction

The cutting and bin packing problem is one of the classic and challenging problems in the field of computer science. This problem has been studied for over 30 years and to this date work is going on to find new improvised approaches and better results. Cutting and packing to this date has ever new applications and especially the variants of bin packing are important to information technology. This problem has many potential applications in different real world industries (like transportation, logistics, Information Technology, etc.). Some of the applications are scheduling television programming, cutting stock problem, cloud computing, truck loading problem and many more. There are different variants of the problem and similarly several approaches for tackling them, a lot of papers have been published relating to this problem. Since bin packing belongs to the class of  $N_p$  hard problems, it is really difficult to come up with a polynomial time algorithm which solves the problem to give an optimal solution. So as a result, approximation algorithms are presented to find the closest possible solution to the optimal.

2D bin packing problem is a generalization of 1D bin packing problem and is an Np-hard problem, too. The potential use of 2D bin packing in many real time and industrial applications is a motivating factor in studying this problem. This 2D bin packing is used in applications like packing items in warehouses and trucks, cutting stock problems (cutting rectangles from sheets of a given size) and many more. These algorithms can be broadly classified into two categories

- Online bin packing Algorithms
- Offline Bin packing Algorithms

*Online bin packing* algorithms pack items in the bin as per the input sequence. These algorithms do not have knowledge of the next items in the input sequence whereas the *offline algorithm* has knowledge of the next item in the input sequence required for bin packing and can possibly arrange them in a particular order before packing the items in the bins.

We are interested in the offline version of the 2D bin packing problem. Some results are known in this field for rectangles of fixed size. Particularly, in 1982 Chung [1] gave an approximation algorithm, a better one was given in 2002 by Caprara [2]. In 2009 Bansal [3] presented a randomized algorithm which improved the previous ones. He also showed that the two-dimensional bin packing problem does not admit an asymptotic polynomial-time approximation scheme.

A. M. Chwatal and S. Pirkwieser [4] in 2011 were solving the two-dimensional bin packing problem with variable bin sizes by greedy randomized adaptive search procedures and variable neighborhood search. In 2011, F. Eisenbrand, D. Palvolgyi and T. Rothvo [5] presented a paper called Bin Packing via Discrepancy of Permutations. A. Layeb and S. Chenche [6] came up with a paper titled A Novel GRASP Algorithm for Solving the Bin Packing Problem which was published in 2012. In the same year G. Perboli, R. Tadei, M. M. Baldi [7] published a paper on the stochastic generalized bin packing problem and along with Crainic they presented another paper on branch-and-price and beam search algorithms for the generalized bin packing problem [8]. This is not the full list of papers on cutting and packing problem, but these papers indicate the amount of work and research performed in this field and emphasize the importance of the considered problem.

We are interested in 2D cutting and packing problem with irregularly shaped objects. In [9] the author presented the description of the two - dimensional modeling program of irregularly shaped objects. The next step of achieving the optimal arrangement of any irregular shape details on the surface is the discretization of geometric models. In this paper a solution to this problem is given. The solution is described in steps. In the second section the circumscribing rectangle with the minimal area for geometric models is found. This brings the problem to the optimal distribution of the rectangles on rectangle. The process of discretization is given in section three.

## 2. Finding the Circumscribing Rectangle with Minimal Area

To solve this task we construct circumscribed rectangles with the sides parallel to the axes of coordinates by rotating the element around its center and determining which of them is the one with the minimal area. Theoretically we have to rotate the element in all possible angles and build circumscribed rectangles, then we have to find the rectangle with minimum area. But one can notice that it is enough to rotate only 0 - 90 degrees, as after every 90 degrees the area is repeated.

The finding of the minimal area is realized by the method of comparing. The first rectangle is considered as the one with minimal area, after we rotate the element in 1 degree and compare the new area of circumscribed rectangle with the previous position and if it is smaller it is accepted as

the rectangle with minimal area, otherwise just passing forward up to 90 degrees. When we find the rectangle with minimal area we rotate the detail back to the corresponding angle.

### 3. Discretization

At this step the circumscribed rectangle must be divided into small squares. The size of this squares depends on the user's decision who must solve the trade-off problem: accuracy of the measurements against speed of the work. On default the factor of discretization equals 30 pixels.

The number of squares is given by the following formulas:

$$N_h = \frac{L + (l - 1)}{l}, \quad (1)$$

$$N_v = \frac{H + (l - 1)}{l},$$

where  $N_h$  and  $N_v$  are, correspondingly, the quantities of horizontal and vertical squares,  $L$  and  $H$  are the width and height of the circumscribed rectangle with the minimal area and  $l$  is the square size or discretization factor. As the factor of the discretization must be a whole number, the obtained values are round up to the nearest integer. The result of the division is given in Figure 1.

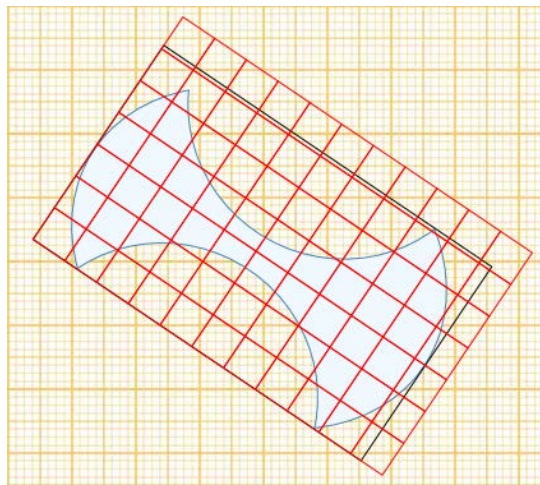


Fig. 1. Circumscribed rectangle is divided into small squares.

The squares that are empty must be removed. The result is given in Figure 2.

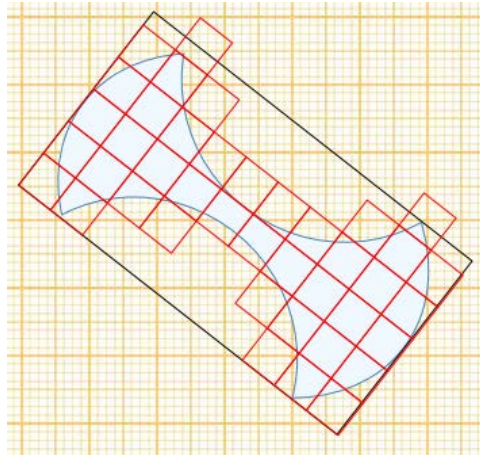


Fig. 2. Empty squares are removed.

The same result for smaller discretization factor (5 pixels) is depicted in Figure 3.

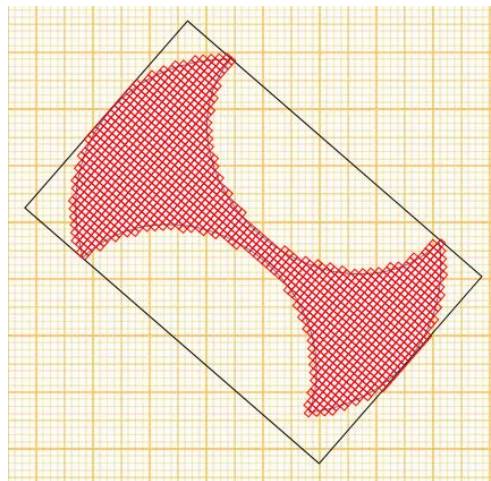
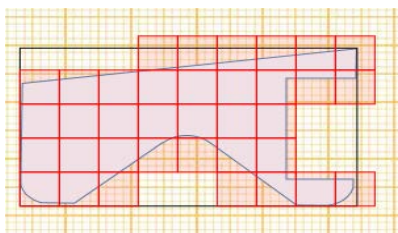


Fig. 3. Small discretization factor.

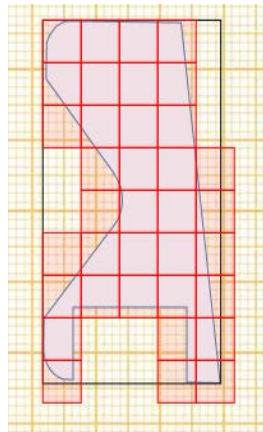
In the final stage to each rectangle we correspond a matrix with 1 for each square and 0 otherwise (fig. 4).



0	0	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1	1

Fig. 4. Corresponding matrix.

This will help to simplify the solution of the next problem of optimal distribution. For example, to rotate the image on 90 degrees the transpose matrix will correspond (fig. 5).



1	1	1	1	0
1	1	1	1	0
1	1	1	1	0
0	1	1	1	1
0	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	0	0	1	1
1	0	0	1	1

Fig. 5. Rotation on 90 degrees.

To restore the image corresponding to each matrix, the number of the detail is stored with the corresponding matrix.

#### 4. Conclusion

In the paper the process of discretization of geometric models is described. First the circumscribing rectangle with the minimal area for geometric models is found to bring the problem to the optimal distribution of the rectangles on rectangle. In the next step the circumscribed rectangle is divided into small squares and the squares that are empty are removed. To simplify the solution of the next problem of optimal distribution each rectangle is associated with a binary matrix.

#### References

- [1] F. Chung, M. Garey and D. Johnson, "On packing two-dimensional bins", *SIAM J. Alg. Disc. Meth.*, vol. 3, no. 1, pp. 66–76, 1982.
- [2] A. Caprara, "Packing 2-dimensional bins in harmony", *Proceedings of the Symposium on Foundations of Computer Science (FOCS)*, pp. 490–499, 2002.
- [3] N. Bansal, A. Caprara and M. Sviridenko, "A new approximation method for set covering problems with applications to multidimensional bin packing", *SIAM J. Comput.* vol. 39, no. 4, pp. 1256–1278, 2009.
- [4] A. M. Chwatal and S. Pirkwieser, "Solving the two-dimensional bin-packing problem with variable bin sizes by greedy randomized adaptive search procedures and variable neighborhood search", *EUROCAST*, vol. 1, pp. 456-463, 2011.
- [5] F. Eisenbrand, D. Palvoelgyi and T. Rothvoss, "Bin packing via discrepancy of permutations", *Symposium on Discrete Algorithms (SODA 2011)*, San Francisco, USA, pp. 22-25, 2011.

- [6] A. Layeb and S. Chenche, “A novel grasp algorithm for solving the bin packing problem”, *International Journal of Information Engineering and Electronic Business (IJIEEB)*, vol. 4, no. 2, pp. 8-14, 2012.
- [7] G. Perboli, R. Tadei and M. M. Baldi, “The stochastic generalized bin packing problem”, *Discrete Applied Mathematics*, vol. 160, no. 7-8, pp. 1291—1297, 2012.
- [8] M. M. Baldi, T. G. Crainic, G. Perboli and R. Tadei, “Branch-and-price and beam search algorithms for the generalized bin packing problem”, *CIRRELT*, Montreal, Canada, pp. 1--18, 2012.
- [9] A. Keyan, “Modeling of irregularly shaped two – dimensional objects”, *Transactions of the IPAA of the NAS RA, Mathematical Problems of Computer Science*, vol.44, pp. 154--160, 2015.

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## Երկրաչափական մոդելների դիսկրետացում

Ա. Քեյան

### Ամփոփում

Մեզ հետաքրքրում է կամայական ուրվագծով երկչափ օբյեկտների ձևման և փաթեթավորման խնդիրը: Նախորդ աշխատանքում հեղինակը ներկայացրել է կամայական ուրվագծով օբյեկտների երկչափ մոդելավորման ծրագրի նկարագրությունը: Կամայական ուրվագծով օբյեկտների մակերևույթի վրա օպտիմալ բաշխման հաջորդ քայլը երկրաչափական մոդելների դիսկրետացումն է: Այս աշխատանքում լուծված է այդ խնդիրը: Նախ գտնում ենք երկրաչափական մոդելին արտագծած ուղղանկյուն նվազագույն մակերեսով խնդիրը հանգեցնելու ուղղանկյուններն ուղղանկյան վրա օպտիմալ բաշխմանը: Հաջորդ քայլում արտագծած ուղղանկյունը բաժանում ենք փոքր քառակուսիների և դատարկ քառակուսիները հեռացնում: Օպտիմալ բաշխման խնդրի լուծումը հեշտացնելու նպատակով յուրաքանչյուր ուղղանկյանը համապատասխանեցվում է երկուական մատրից:

## Дискретизация геометрических моделей

А. Кеян

### **Аннотация**

Нас интересует задача 2D раскроя и упаковки объектов неправильной формы. В предыдущей работе автор представил описание программы двухмерного моделирования объектов неправильной формы. Следующим шагом оптимального распределения деталей неправильной формы на поверхности является дискретизация геометрических моделей. В данной работе решена эта задача. Сначала находится описанный прямоугольник с минимальной площадью для геометрической модели, чтобы привести к проблеме оптимального распределения прямоугольников на прямоугольнике. На следующем шаге описанный прямоугольник делится на небольшие квадраты и пустые квадраты удаляются. Чтобы упростить решение следующей задачи оптимального распределения каждому прямоугольнику ставится в соответствие бинарная матрица.