

Statistical Tests for MIXMAX Pseudorandom Number Generator

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Abstract

The Pseudo-Random Number Generators (PRNGs) are key tools in Monte Carlo simulations. More recently, the MIXMAX PRNG has been included in ROOT and Class Library for High Energy Physics (CLHEP) software packages and claims to be a state of the art generator due to its long period, high performance and good statistical properties. In this paper the various statistical tests for MIXMAX are performed. The results compared with those obtained from other PRNGs, e.g., Mersenne Twister, Ranlux, LCG reveal better qualities for MIXMAX in generating random numbers. The Mersenne Twister is by far the most widely used PRNG in many software packages including packages in High Energy Physics (HEP), however, the results show that MIXMAX is not inferior to Mersenne Twister.

Keywords: MIXMAX, Statistical tests, MCMC.

1. Introduction

In recent years, there is a growing interest on PRNGs in different branches of physics and not only. A good PRNG is important to have guaranteed results of Monte Carlo(MC) methods. There are many software packages for MC simulations where PRNGs are the central components. Among these packages one can mention the Geant4/CLHEP[1], a widely used simulation toolkit in HEP for modeling the passage of elementary particles through matter, also used for medical and space science simulations.

PRNGs are also crucial in Markov Chain Monte Carlo (MCMC) methods which are used for sampling from desired probability distribution by constructing Markov chain on state space whose stationary distribution is of interest[2, 3, 4]. Uniform PRNGs play a central role in constructing such Markov Chains. Most of MCMC algorithms are developed within random walk models. A widely used example of random walk Monte Carlo method is MetropolisHastings algorithm[3, 4, 5, 6] which is also included in the list of the top 10 algorithms[7]. MCMC methods are mainly used for sampling from large dimensional spaces and computing multidimensional integrals. For example, in statistical mechanics, one needs to compute thermal averages of quantities, such as the total energy, magnetization, etc. by performing multidimensional integration or summation over configuration space. however,, the total number of configurations can be very large, e.g., in 3-dimensional Ising model the number of spin configurations with particles at n^3 lattice sites is 2^{n^3} . In thermodynamic

equilibrium the probabilities of occurring each configuration is represented by Boltzmann distribution. Thereby having samples drawn from Boltzmann distribution one can compute expectation values of thermodynamic quantities.

The necessity to have large amounts of simulated data imposes a strict requirements on PRNGs, such as statistical properties of generated numbers, swiftness in number generation, replicability, lengthiness of generated random cycle and independence of produced random numbers. To address these challenges the renewed version of MIXMAX PRNG[8, 9] based on Anosov C-systems and Kolmogorov K-systems has been introduced in [10, 11, 12]. The MIXMAX is matrix-recursive PRNG and it has been shown that the properties of the MIXMAX generator is improved with increasing the size N of MIXMAX matrix[12]. The period of MIXMAX is also increased with increasing N and it can reach up to 10^{57824} , note that the period of commonly used version of Mersenne Twister based on Mersenne prime has the period of $2^{19937} - 1$.

While having a long period, however, statistical properties and time characteristics of PRNGs are crucial to consider a generator "good" or "bad". In this paper we will present the results of the statistical tests performed with the matrix size of $N = 256$ which is considered to be a default dimension of MIXMAX matrix with flexibility to be further increased.

2. Visual Demonstration

We can reveal the defect of uniform PRNGs simply plotting random points in high-dimensional Euclidean space, if these points form a lattice structure then to a first approximation we can say that PRNG has defects in generating random points since the space is not filled uniformly. The Fig.1 shows the comparison of MIXMAX with the Linear Congruential Generator(LCG), which is known to be defective PRNG. In contrast to LCG, MIXMAX does not form a lattice structure. We obtain these figures by generating two $U(0, 1)$ random number sequences and assigning a point in two-dimensional space.

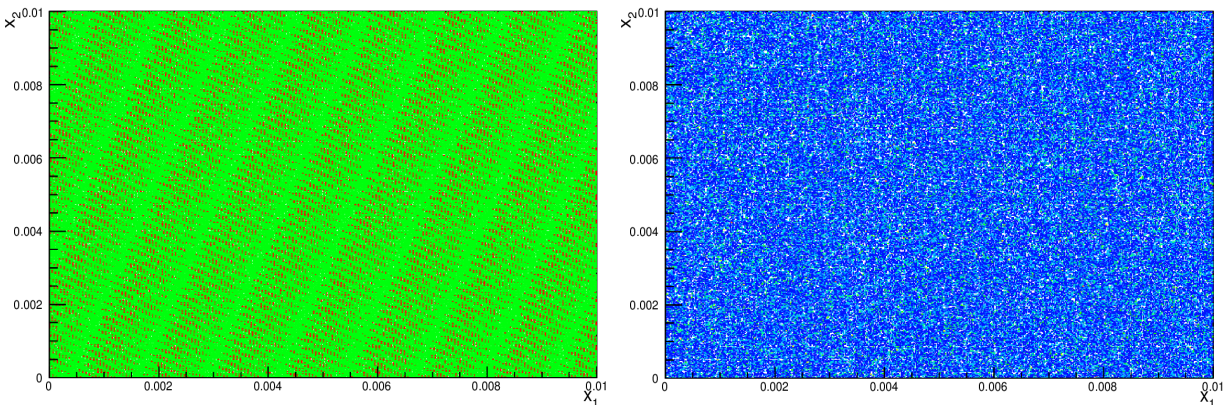


Fig. 1. Random points in two-dimensional space generated by LCG(left) and MIXMAX(right).

3. Statistical Testing with TestU01

Most of PRNG algorithms produce numbers uniformly distributed in the interval of $[0, 1]$, hence, PRNGs should pass statistical tests of uniformity. Many empirical statistical testing

packages implement tests for these purposes, some of which are [13, 16, 14, 15]. Most of the statistical tests implemented in these packages are discussed in Knuth's book [17], e.g., the package [14] implements mainly the tests of Knuth. Currently, one of the well-known tools for statistical testing is TestU01 software library which provides implementations of the empirical statistical tests for uniform PRNGs. It contains more than 160 different empirical tests and offers several batteries of tests including the most powerful one, i.e., the 'Big Crush'. When a specific statistical test is applied to random numbers produced by PRNG the p -value of the test is printed as a measure of deviation from null-hypothesis, which in our case is a uniform distribution of random numbers. In comparison with other libraries TestU01 is more flexible and efficient, and it can deal with larger sample sizes and has a wider range of statistical tests than the other libraries.

In Table 1, the outcome of TestU01 BigCrush suite applied on MIXMAX, Mersenne Twister and LCG is stored by using 64-bit computer with *Intel Core i3 -4150* processor of clock speed $3.50 \times 4 GHz$.

Table 1: TestU01 BigCrush suite results.

PRNG	Total CPU time	BigCrush	Failed test'(s) p value
MIXMAX	<i>2h 43m 51s</i>	<i>All tests were passed</i>	—
Mersenne Twister	<i>3h 19m 27s</i>	3	$0.9990, 1 - 10^{-15}$
LCG	<i>3h 30m 33s</i>	22	$< 10^{-300}$

As we can see from the table the MIXMAX passes the same test suite faster than Mersenne Twister and does not fail any test. TestU01 test suite has been applied to Ranlux PRNG with its modifications Ranlux24, Ranlux48. It is observed that Ranlux, though having good statistical properties is very slow at generating random numbers. Comparing with MIXMAX, Ranlux24 is 10 times slower and Ranlux48 is 17 times slower. This fact makes it not convenient for the use in generation of large amount of random numbers.

4. Kolmogorov-Smirnov Tests

Kolmogorov-Smirnov (K-S) test is one of the powerful tools that can be used to examine the statistical features of PRNGs.

Though one-dimensional (1D) K-S test is already implemented in TestU01, we perform K-S test independently for various parameters of sample size (n) and extract the distribution of K-S test statistic. The idea behind the test is to calculate the maximum distance between the expected Cumulative Distribution Function(CDF) $F(x)$, $F(x) = Pr(X \leq x)$ and measured or Empirical Cumulative Distribution Function(ECDF) $F_n(x)$ of n data points

$$F_n(x) = \frac{1}{n}(\text{number of } x_i \leq x). \quad (1)$$

The null hypothesis H_0 is whether the sample of n random numbers comes from the expected distribution $F(x)$ or not. If data comes from $F(x)$, then the strong law of large numbers provides $F_n(x) \rightarrow F(x)$, as $n \rightarrow \infty$. The latter is strengthened by the Glivenko-Canteli (G-C) theorem[18], which states that under H_0 hypothesis

$$Pr(\lim_{n \rightarrow \infty} \sup_x |F_n(x) - F(x)| = 0) = 1. \quad (2)$$

Hence the difference between CDFs can be used as a measure of agreement between a data and a given distribution. There are several statistical tests based on (G-C) theorem known as Cramer-von Mises tests [19, 20]. The key feature of tests based on G-C theorem is that their distributions are independent of the hypothesized models under H_0 when data sample is large. The one dimensional(1D) K-S test is defined as follows:

$$D_n = \sup_x |F_n(x) - F(x)|. \quad (3)$$

Under null hypothesis the distribution of $\sqrt{n} \cdot D_n$ converges to Kolmogorov distribution for sufficiently large n when $F(x)$ is continuous[21]

$$\lim_{n \rightarrow \infty} Pr(\sqrt{n} \cdot D_n \leq x) \equiv K(x) = 1 - 2 \sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2 x^2}. \quad (4)$$

It is of interest to note that for small values of n Kolmogorov distribution is not adequate, but there is a way to compute p - value for randomly produced D_n [21]. In Fig. 4. the normalized histogram of $\sqrt{n} \cdot D_n$ data points for MIXMAX and Mersenne Twister is presented and compared with Probability Density Function (PDF) of Kolmogorov distribution: $f(x) = K'(x)$

$$f(x) = 8x \sum_{i=1}^{\infty} (-1)^{i-1} i^2 e^{-2i^2 x^2}. \quad (5)$$

The histograms in Fig.(2-7) are normalized to unity dividing each bin entry by the product of sample size and bin width ($n \cdot width$). Visual comparison shows that under H_0 the distribution of $\sqrt{n} \cdot D_n$ follows the PDF of Kolmogorov distribution. Due to fast convergence of the series of partial sums in Eq.(4,5) it suffices to take a limited number of terms in the sum, e.g., the first 100 terms are enough.

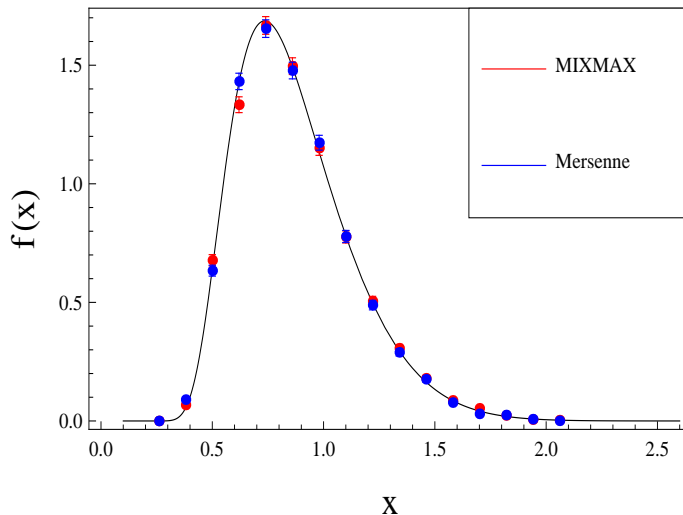


Fig. 2. Distribution of $\sqrt{n} \cdot D_n$ for MIXMAX and Mersenne Twister. The size of a samples is $n = 10^8$ and the number of different replicas is 10^4 . The black curve is the PDF of Kolmogorov distribution.

Two-level tests can be used on K-S test to give evidence of visual coincidence on Fig. 4.. For this purpose the chi-square test(next section) is applied. It has been checked that both distributions agree with theoretical expectation (black curve) in 95% Confidence Level (CL).

In multidimensional K-S test one has d -dimensional data ($d \geq 2$) and to test H_0 it is needed to compare d -variate ECDF with the hypothetical d -variate CDF. The complication in multidimensional case is caused by the ambiguity in definition of the CDF since there are $2^d - 1$ independent ways of defining CDFs. There have been proposed different ways to calculate the multidimensional K-S statistic [22],[23]. In [22] four quadrants around all combinations (x_i, x_j) of data points are considered and D is taken as the maximum of 4 differences between CDFs over all quadrants. Therefore, this idea makes the test statistic independent of ordering the data. The number of all pairs (x_i, x_j) for N points is equal to N^2 , therefore, to calculate D_n one needs to compute the differences between CDFs in $3N^2$ quadrants (the probability for fourth quadrant is found from normalization). This method suffers from the computing time when N is large. In [23], it is proposed to consider only the observed points rather than all combinations, thereby reducing the computational time by computing the differences for $3N$ quadrants only. It is possible to compute D_n with computationally higher efficiency introducing a binning technique applied to a continuous multidimensional data, i.e., discretizing the data space. The idea of binning technique is discussed in [24]. In [25] the algorithm for 2D K-S test is presented when only one CDF from all the possible configurations is taken into account. As a result of it, the procedure used to compute D evaluating the difference of CDFs is reduced to a small number of data points. In our studies we have extended the standard definition of one dimensional cumulative distribution to its two dimensional “analogue” and computed K-S statistic using the algorithm presented in [25] (see Fig. 4).

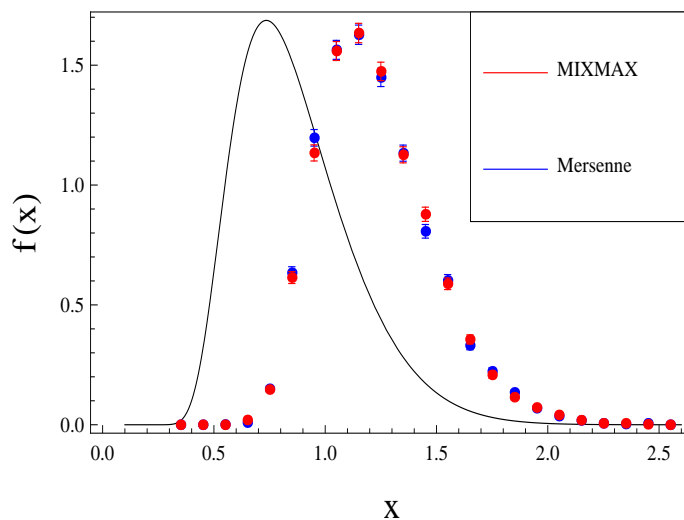


Fig. 3. Distribution of $\sqrt{n} \cdot D_n$ for MIXMAX and Mersenne Twister for 2-dimensional case. The size of a samples is $n = 10^3$, note that n is the total number of random points in 2-dimensional space, i.e., 10^6 numbers are generated by PRNGs, the number of different replicas is 10^4 . The black curve is the PDF of Kolmogorov distribution. The shape of histogram shows a clear shift from the Kolmogorov distribution. MIXMAX and Mersenne Twister that have been used in these studies give almost the same distributions for $\sqrt{n} \cdot D_n$ which seems to be independent from the dimension of the Kolmogorov-Smirnov test.

Three-dimensional extension of K-S test is presented in [26], where 8 CDFs are considered, and using MC techniques the table of critical values are also presented in this paper.

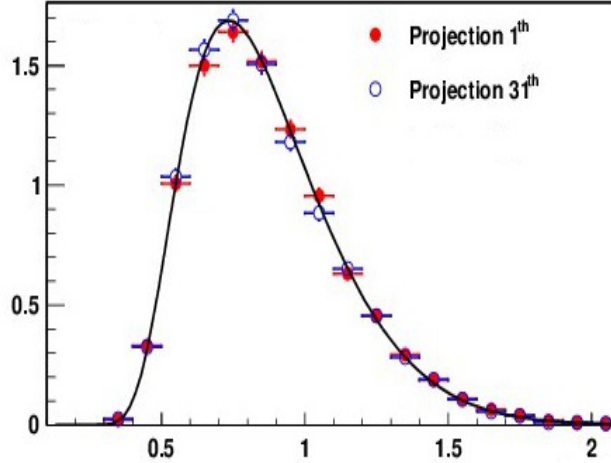


Fig. 4. Distribution of $\sqrt{n} \cdot D_n$ for the 1st and 31st projections of MIXMAX RNG in comparison with the theoretical expectation (black curve). The sample size is $n = 10^8$ and for each projection 10^4 different replicas are generated.

To detect possible non-uniformities in the multidimensional random sequences of MIXMAX PRNG, an arbitrary selected projection has been checked via Kolmogorov-Smirnov test and the results are compared with those of the first projection. In Fig. 4., the probability density distributions of Kolmogorov-Smirnov statistic are presented for the 1st and the 31st projections and the comparison is provided with the expected distribution.

5. Chi-Square Tests

The chi-square χ^2 test is one of the famous statistical tests which is found in many applications when one deals with grouped or binned data. The Chi-Square test is applied to categorical sample distributions unlike the K-S test which using each random point compares the continuous sample distributions with the hypothesized ones. The χ^2 statistic has the following form [17, 27]:

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}, \quad (6)$$

where O_i is the observed number of data points in i th bin and $E_i = np_i$ is the expected number of data points falling into i th bin, here p_i is the probability that observation falls into i th bin. To apply the test to PRNGs $[0,1]$ interval is divided into k bins and the χ^2 statistic is computed noting that $p_i = 1/k$. If H_0 is true then statistics defined in (6) computed for random samples follows chi-squared distribution with $\nu = (k - 1)$ degrees of freedom

$$g_\nu(y) = \frac{2^{-\frac{\nu}{2}} e^{-\frac{y}{2}} y^{\frac{\nu}{2}-1}}{\Gamma(\frac{\nu}{2})}, \quad (7)$$

where $\Gamma(\nu)$ is a gamma function. It is useful to introduce a new random variable $x = \frac{y}{\nu}$ and consider the PDF of x denoted as $f_\nu(x)$. This enables to get rid of small numbers in PDFs when ν is big. Since $1 = \int f_\nu(x)dx = \int g_\nu(y)dy$ it follows that

$$f_\nu(x) = \nu g_\nu(\nu x) = \frac{2^{-\frac{\nu}{2}} \nu^{\frac{\nu}{2}} e^{-\frac{x}{2\nu}} x^{\frac{\nu}{2}-1}}{\Gamma(\frac{\nu}{2})}. \quad (8)$$

The new random variable introduced in (8) is called a reduced chi-square. The distribution of the reduced chi-square for MIXMAX and Mersenne Twister is shown in Fig. 5. in comparison with the theoretical expectation.

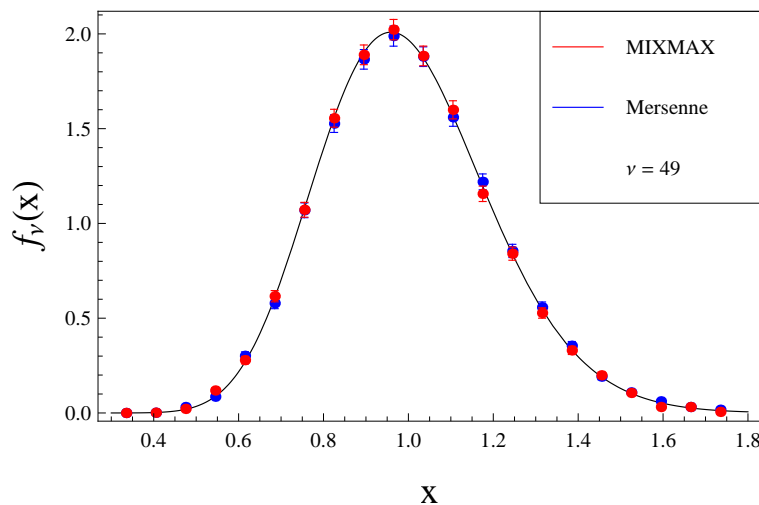


Fig. 5. Comparing density histogram of reduced chi-square for MIXMAX and Mersenne Twister with the PDF in (8). The size of a samples is $n = 10^6$ and the number of different replicas is 10^4 .

6. Serial Tests

The serial test also known as a chi-square test of independence is a multidimensional analogue of the chi-square test which checks the independence between two or more random variables [17, 28]. When the serial test is applied to PRNGs one divides the random sequence into groups of non-overlapping d -tuples $(x_{id}, x_{id+1}, \dots, x_{id+k-1})$, where $i = 1, 2, \dots, \frac{n}{d}$, hence the elements of d -tuple are considered as realizations of d random variables and the relationship between them is of interest. If x_i s are $U(0, 1)$ random variables then k -tuples are uniformly distributed in $[0, 1]^d$. To check this each dimension of unit hypercube is divided into k bins and the data of d -tuples is binned into $[0, 1]^d$. Now chi-square statistic (6) is applied to this data comparing the number of observations falling in each sub-hypercube with theoretical expectation: $E_{i,j,\dots,d} = np_{i,j,\dots,d}$, where the joint probability $p_{i,j,\dots,d}$ of d -dimensional data point to fall into (i, j, \dots, d) sub-hypercube is the product of probabilities of each individual coordinate to fall into an appropriate bin, which is the condition of independence.

$$p_{i,j,\dots,d} = \prod_{n=1}^d p_n = \left(\frac{1}{k}\right)^d. \quad (9)$$

Unlike the non-overlapping tuples, the overlapping d -tuples of random sequence fall on neighboring parallel planes. The largest distance between the adjacent parallel hyperplanes is called a spectral test statistic [17, 29, 30, 31, 32, 33]. If the largest distance is small then it implies that overlapping d -tuples are more uniformly distributed in unit hypercube, therefore, PRNG is considered good.

Fig. 6. shows $d = 3$ and $d = 5$ dimensional cases of serial test, where each dimension is divided into $k = 10$ bins and the histogram of the reduced chi-square test statistics is compared with the distribution of (8) with appropriate degrees of freedom. The reduced chi-square distribution reveals no significant distinction between MIXMAX and Mersenne Twister. Note that the serial test here is applied to a single random stream and measures the correlations between adjacent random tuples. This test can be also applied to different streams to check the independence between them.

7. Parallel Streams of MIXMAX

All tests described in previous sections use one stream generated by PRNG. however,, in multiprocessor stochastic computations it is important to have uniformly distributed and statistically independent simultaneous random streams partitioned across the processors [34, 35, 36, 37, 38]. Different parallelisation approaches of PRNGs have been studied in literature [39, 40, 41, 38]. One trivial technique for parallelisation is to take random seeds on each processor, but since every PRNG has a finite number of states, one should be careful in order to avoid possible overlapping between different streams. MIXMAX has very large state space, therefore, even taking random seeds on each processor does not affect the independence between multiple streams. Another approach is to take a single sequence and partition it into different processors. MIXMAX provides a skipping-ahead algorithm which enables to skip forward by large amount of numbers in sequence, this technique guarantees the non-collision of partitioned streams [39]. The check for randomness of each individual stream can be done via the standard chi-square or K-S tests. The test of independence of multiple streams can be done via parallel version of serial test simply forming d -tuples of random numbers taken from each of d streams at a time. however,, it is not practical to test empirically all random streams when the period is very large, different techniques for testing parallel streams can be found in [38].

The serial test up to dimensions $d = 7$ has been performed and it is observed that multiple streams of MIXMAX are statistically independent which is also guaranteed by underlying theory of MIXMAX.

One can analyze the independence and uniformity of parallel streams using the fact that the sum of n independent $U(0, 1)$ random variables follow the IrwinHall distribution of order n [42].

Under H_0 if each stream of PRNG is generated from a uniform distribution then the random sequence resulted from the element-wise addition of multiple streams has IrwinHall distribution. The IrwinHall distribution has the following form:

$$f_n(x) = \frac{1}{2(n-1)!} \sum_{i=1}^n (-1)^i \binom{n}{i} (x-i)^{n-1} \text{sgn}(x-i), \quad (10)$$

where $\text{sgn}(x-i)$ is a *sign* function. When $n = 2$, then (10) reduces to the well known

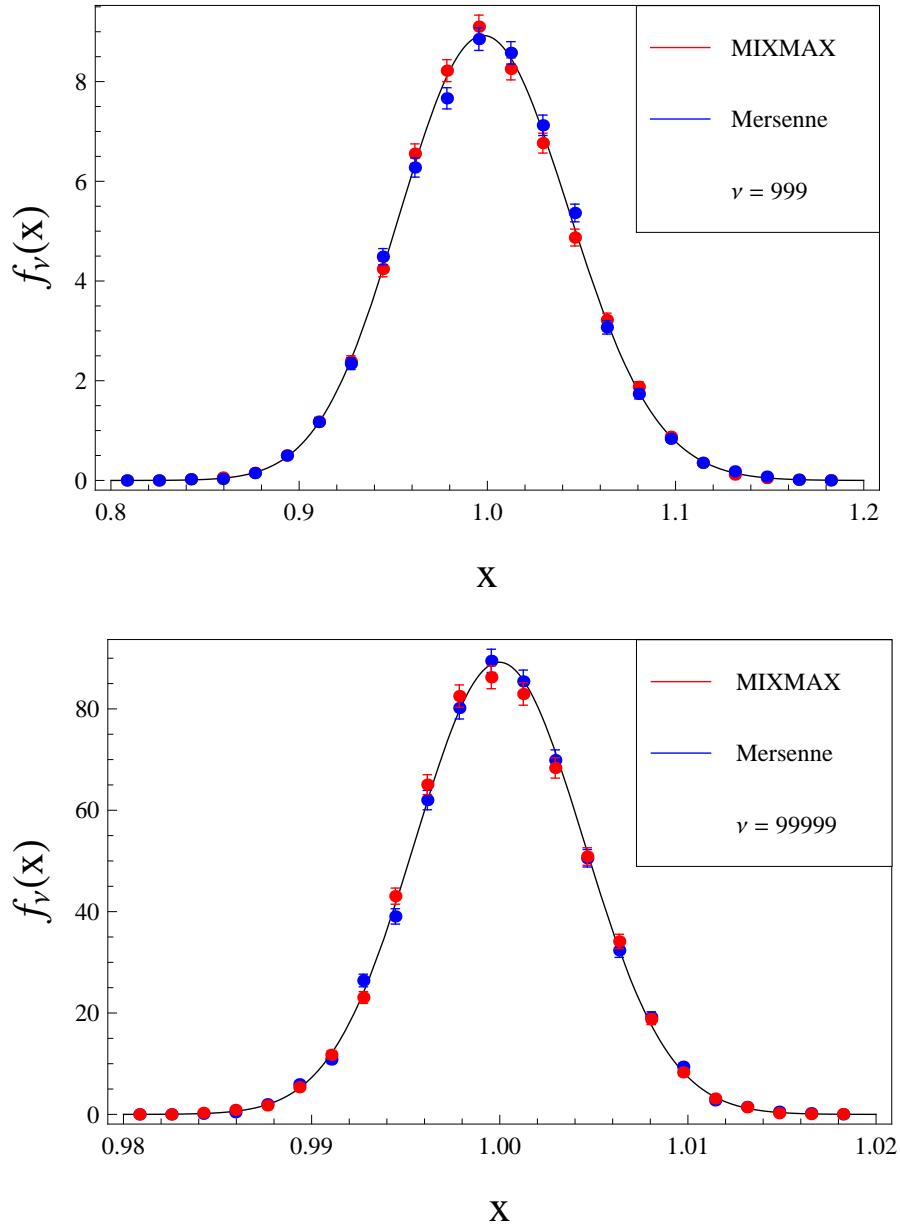


Fig. 6. Comparing density histogram of reduced chi-square with the test distribution (8) for 3-dimensional(up) and 5-dimensional(down) cases.

triangular distribution

$$f_2(x) = \begin{cases} x, & 0 \leq x < 1, \\ 2 - x, & 1 \leq x \leq 2. \end{cases} \quad (11)$$

In Fig. 7. visual comparison of the data with IrwinHall distribution is shown, where up to 15 random streams are taken to form the sum. To check visual consistency of the histogram and model prediction in Fig.(7.) a chi square test is applied for comparison with Irwin-Hall distribution. Table 2 represents p -values of chi square statistic.

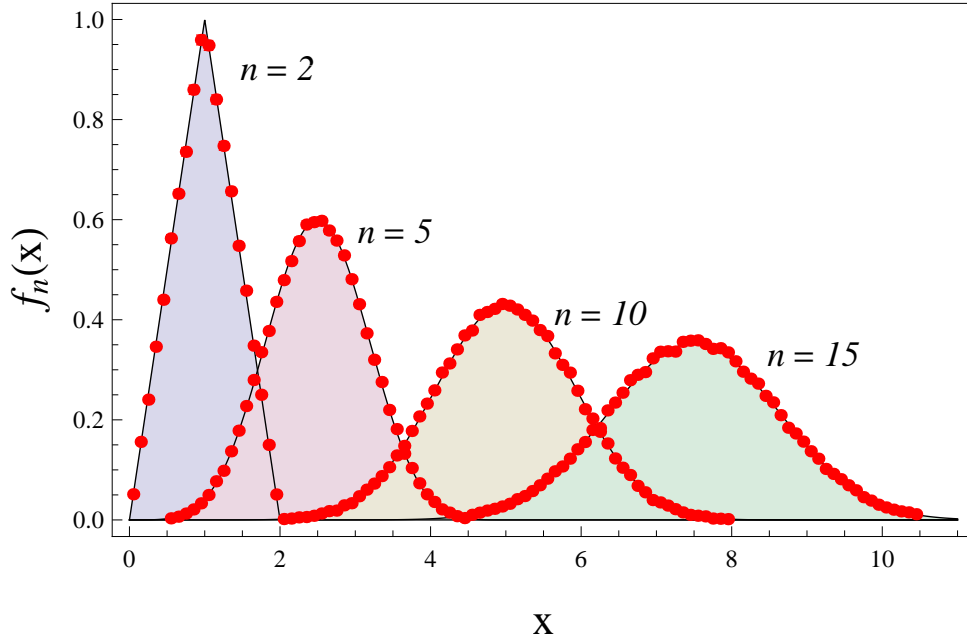


Fig. 7. The distribution of Irwin-Hall of order (2,5,10,15) compared with density histogram of data.

Table 2: p -values from chi-square test.

Test	n = 2	n = 5	n = 10	n = 15
p-value	0.76	0.97	0.39	0.37

8. Conclusion

This paper presents the study of newly released MIXMAX PRNG. The various statistical tests including a very high quality TestU01 have been used to check the quality of MIXMAX compared with other generators, mainly with Mersenne Twister. The results show that MIXMAX is not inferior to Mersenne Twister and even better in the sense of speed and period.

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Ստատիստիկական թեստեր պսևդո-պատահական թվերի գեներատոր MIXMAX-ի համար

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Անփոփում

Պսևդո-պատահական թվերի գեներատորները առանցքային գործիք են հանդիսանում Մոնտե Կարլո մեթոդով մոդելավորման համար: Վերջերս նոր "MIXMAX" գեներատորը ընդգրկվեց ROOT և Class Library for High Energy Physics (CLHEP) ծրագրային փաթեթների մեջ երկար պարբերության, արագագործության և լավ ստատիստիկ հատկությունների շնորհիվ: Ներկայացվող հոդվածում ուսումնասիրվել են "MIXMAX"-ի ստատիստիկ հատկությունները տարբեր ստատիստիկական թեստերով: Արդյունքները համեմատվել են տարբեր գեներատորներից ստացված արդյունքների հետ, օրինակ՝ "Mersenne Twister"-ի, "Ranlux"-ի և "LCG"-ի: Յուրյց է տրվել որ "MIXMAX"-ն ունի ավելի լավ հատկություններ պատահական թվերի գեներացման համար: "Mersenne Twister" գեներատորը հանդիսանում է ամենատարածված գեներատորը շատ ծրագրային փաթեթներում, սակայն ներկայացված արդյունքները ցույց են տալիս, որ "MIXMAX"-ը ոչ միայն չի զիջում "Mersenne Twister"-ին, այլ նաև գերազանցում է նրան:

Статистические тесты для генератора псевдослучайных чисел MIXMAX

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Аннотация

Генераторы псевдо-случайных чисел (ГПСЧ) являются ключевыми инструментами моделирования по методу Монте Карло. Недавно новый ГПСЧ "MIXMAX" был включен в пакеты программного Обеспечения ROOT и Class Library for High Energy Physics (CLHEP) ввиду того, что этот генератор признан одним из лучших существующих ГПСЧ по длинному периоду, высокой производительности, а также отличным статистическим свойствам. В предлагаемой статье приводятся результаты различных статистических тестов для проверки генератора "MIXMAX". Проведено сравнение характеристик "MIXMAX" с другими ГПСЧ, например с "Mersenne Twister", "Ranlux" и "LCG", показано, что "MIXMAX" обладает лучшими свойствами для генерации случайных чисел. Генератор "Mersenne Twister" является одним из самых используемых во многих приложениях, включая пакеты для физики высоких энергий, однако приведенные результаты показывают, что "MIXMAX" ни в чем не уступает и даже превосходит "Mersenne Twister".