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A New Method of Solving Diophantine Equation $a^3 + b^3 + c^3 = d$

Armen A. Avagyan

Armenian State Pedagogical University
e-mail: avagyana73@gmail.com

Abstract

The article is dedicated to the famous Diophantine equation of the form $a^3+b^3+c^3 = d$. We solve this problem in some particular cases. Using the package Mathematica 11 we find an efficient algorithm to solve this problem. This algorithm is simpler and uses a significantly smaller number of operations than the other known algorithms for solving these equations.

Keywords: Diophantine equations, the sum of three cubes, Parametric solutions, Polynomial identities.

1. Introduction

Fermat's equation for odd exponents n asks for three integers, each with an absolute value greater than 0, such that the sum of their $n-$th powers is zero. A related problem is to find three integers, each with an absolute value greater than the $n-$th root of k, such that the sum of their $n-$th powers equals k.

For example, determine the integers a, b, c, with $|a|, |b|, |c| > 1$ such that

$$a^3 + b^3 + c^3 = d. \tag{1}$$

This has infinitely many solutions because of the identity

$$\left(1 - 9m^3\right)^3 + \left(9m^4\right)^3 + \left(-9m^4 + 3m\right)^3 = 1 \ . \tag{2}$$

However, there are other solutions as well. Are there any other identities that give a different 1-parameter family of solutions?

Is every solution of (1) a member of a family like this? In general, it is known that there is no finite method for determining whether a given Diophantine equation has solutions. However, it is an open problem, whether there is a general method for determining if a given Diophantine equation has "algebraic" solutions, i.e., an algebraic identity like the one above that gives an infinite family of solutions. More specifically, is there a proposition, that only equations of $genus < 2$ can have an algebraic solution?

It may be worth mentioning that the complete rational-solution of the equation $a^3 + b^3 + c^3 = t^3$ is known, and is given by

$$a = q\left[1 - (x - 3y)\left(x^2 + 3y^2\right)\right],$$

$$b = -q\left[1-(x+3y)\left(x^2+3y^2\right)\right],$$

$$c = q\left[\left(x^2+3y^2\right)^2-(x+3y)\right],$$

$$t = q\left[\left(x^2+3y^2\right)^2-(x-3y)\right],$$

where q, x, y are any rational numbers.

Thus, if we set q equal to the inverse of $\left[(x^2+3y^2)^2-(x-3y)\right]$ we have rational solutions of (1). However, I think the problem of finding the integer-solutions is more difficult. If t is allowed to be any integer (not just 1) then Ramanujan gave the integer solutions as

$$\begin{aligned} a &= 3n^2-5\text{nm}-5m^2 \\ b &= 4n^2-4\text{nm}+6m^2 \\ c &= 5n^2-5\text{nm}-3m^2 \\ t &= 6n^2-4\text{nm}-4m^2 \end{aligned}$$

This occasionally gives a solution of equation (1) (with appropriate changes in sign), as in the following cases

n	m	a	b	c
1	-1	1	2	-2
1	-2	9	10	-12
5	-12	-135	-138	172
19	-8	-791	-812	1010
46	-109	11161	11468	-14258
73	-173	65601	67402	-83802
419	-993	-951690	-926271	1183258

However, this does not cover all the solutions given by (2). By the way, the equation $a^3+b^3+c^3=1$ has algebraic solutions [1], [2] other than (2). There are known to be infinitely many algebraic solutions, for instance:

$$(1-9t^3+648t^6+3888t^9)^3+(-135t^4+3888t^{10})^3+(3t-81t^4-1296t^7-3888t^{10})^3=1$$

However, it is not known whether every solution of the equation lies in some family of solutions with an algebraic parameterization.

Interestingly, note, that if you replace 1 by 2, then again there is a parametric solution:

$$\left(6t^3+1\right)^3-\left(6t^3-1\right)^3+\left(6t^2\right)^3=2 \tag{3}$$

and again this does not cover all the known integer solutions. Note, that precisely one solution is known that is not given by (3) (see [1]):

$$1214928^3+3480205^3-3528875^3=2.$$

It is evidently not known until today, if there are any other algebraic solutions besides the one noted above.

For $d>2$ Kenji Koyama [3] has generated a large table of integer solutions of $a^3+b^3+c^3=d$ for noncubes d in the range $1 \le d \le 1000$ and $|a| \le |b| \le |c| \le 2^{21}-1$

consists of two tables: Table 1 (55 pages) contains the integer solutions, sorted by d, and Table 2 (2 pages) lists the number of primitive solutions found for each d in the search range.

In general, it seems to be a difficult problem to characterize all the solutions of $a^3+b^3+c^3=d$ for some arbitrary integer $d>2$. In particular, the question of whether all integer solutions are given by an algebraic identity seems both difficult and interesting.

Nevertheless, for instance, in the case of $d=3$, there is still no solution known apart from the obvious ones: (1, 1, 1), (4, 4,5), (4,5, 4), and (5, 4, 4). For $d=30$, the first solution was found by N. Elkies and his coworkers in 2000 [5]. It is interesting, that in 1992, D. R. Heath-Brown [6] made a prediction on the density of the solutions for $d=30$ without knowing any solution explicitly. Over the years, a number of algorithms have been developed in order to attack the general problem. Concerning the various approaches, an excellent overview, invented before 2000, was given in [7], which was published in 2007. Historically, the first algorithm, which has a complexity of $O\left(B^{1+\epsilon}\right)$ for a search bound of B is the method by R. Heath-Brown [8].

Return the interested representations

$$d=a^3+b^3+c^3 \tag{4}$$

of various integers d as sums of three cubes.

The cubic residues with respect to module 9 are: 0, 1, 8, thus, it follows by inspection of cases that for every integer solution to (4) we obtain $d\neq 6\pm 4(\mathrm{mod}9)$. Any given solution can be written in one of the following forms for non-negative a,b,c :

$$|d|=a^3+b^3+c^3 \text{ or } |d|=a^3+b^3-c^3 \text{ or } |d|=a^3-b^3-c^3$$

Therefore, it suffices to consider non-negative solutions to the equations $a^3+b^3=c^3\pm d$ and $a^3+b^3+c^3=d$ (for $d=0$ it is a case of Fermats theorem that there are no integer solutions).

In practice, we need to search for primitive solutions, i.e., $GCD(a,b,c)$ is not divisible by d, since the non-primitive solutions for fixed n are routinely obtained from the primitive solutions for their divisors.

Considering $d=m^3, d=m^{12}$ and $d=2m^9$ type values of d, and multiplying both sides of (3) by m^9, after applying the change of variable tt/m, one can obtain the more general solution

$$\left(6t^3+m^3\right)^3-\left(6t^3-m^3\right)^3-\left(6t^2\right)^3=2m^9 \tag{5}$$

which is primitive for $GCD(6t,m)=1$. If $GCD(6t,m)>1$, then dividing (5) by $(GCD(6t^3,m^3))^3$ gives a primitive solution. For $l,k\geq 1$ the solutions are:

$$(3t^3+2^{3l-1}m^3)^3-(3t^3-2^{3l-1}m^3)^3-(2^l3mt^2)^3=2^{9l-2}m^9 \tag{6}$$

$$(2t^3+3^{3k-1}m^3)^3-(2t^3-3^{3k-1}m^3)^3-(23^kmt^2)^3=23^{9k-3}m^9 \tag{7}$$

$$(t^3+2^{3l-1}3^{3k-1}m^3)^3-(t^3-2^{3l-1}3^{3k-1}m^3)^3-(2^l3^kmt^2)^3=2^{9l-2}3^{9k-3}m^9 \tag{8}$$

Note, that they are primitive for $GCD(3t,2m)=1$, $GCD(2t,3m)=1$ and $GCD(t,6m)=1$, respectively.

The last equations give polynomial families for n = 2, 128, 1458, 65536, 93312, 3906250, 28697814, etc.

An analogous procedure may be applied for 3 to obtain families of solutions for numbers of the form m^{12}. Multiplying both sides by m^{12} and applying the transformation t/m, we will get

$$\left(9mt^3 + m^4\right)^3 - \left(9t^4 + 3mt\right)^3 + \left(9t^4\right)^3 = m^{12} \tag{9}$$

which is primitive for $GCD(t, 3m) = 1$. In particular, for $3 - m$ and $k1$,

$$\left(3^k mt^3 + 3^{4k-2}m^4\right)^3 - \left(t^4 + 3^{3k-1}lm^3t\right)^3 + \left(t^4\right)^3 = 3^{12k-6}m^{12} \tag{10}$$

is primitive for $GCD(t, 3m) = 1$. Equations (9) and (10) give families of solutions for n = 1, 729, 4096, 2985984, 16777216, 244140625, 387420489,etc.

2. New Method and Results

Considering the more generalized problem of the sum of three cubes, we are seeking $P_1(y), P_2(y), P_3(y)$ with the highest possible degree polynomials and $Q(y)$ with the lowest possible degree polynomial, such as

$$P_1^3(y) + P_2^3(y) + P_3^3(y) = Q(y)$$

Actually, the solution of this problem has a close relation with the above trivial problem, since the case of $degQ(y) = 0$ coincides with our problem. Nevertheless, the estimation of possibility of minimization of $degQ(y)$ itself is also an interesting problem.

Result 1: The first result of this paper is devoted to the case of degrees (8, 8, 6). We search the desired polynomials within the class of polynomials of the form

$$\left(ax^8 + bx^5 + cx^2\right)^3 - \left(ax^8 + b_1x^5 + c_1x^2\right)^3 - \left(Ax^6 + Bx^3 + C\right)^3 \tag{11}$$

First of all, we expand it

$$-C^3 - 3BC^2x^3 + \left(c^3 - 3B^2C - 3AC^2 - c_1^3\right)x^6 + \left(-B^3 + 3bc^2 - 6ABC - 3b_1c_1^2\right)x^9$$

$$+\left(-3AB^2 + 3b^2c + 3ac^2 - 3A^2C - 3b_1^2c_1 - 3ac_1^2\right)x^{12} + \left(b^3 - 3A^2B - b_1^3 + 6abc - 6ab_1c_1\right)x^{15}$$

$$+\left(-A^3 + 3ab^2 - 3ab_1^2 + 3a^2c - 3a^2c_1\right)x^{18} + \left(3a^2b - 3a^2b_1\right)x^{21}$$

Further,we take $b_1 = b$, $c_1 = \frac{-A^3+3a^2c}{3a^2}$, $B = \frac{2Ab}{3a}$, $C = \frac{-A^4-aAb^2+6a^2Ac}{9a^3}$ and get the following form:

$$\frac{A^{12}}{729a^9} + \frac{A^9b^2}{243a^8} + \frac{A^6b^4}{243a^7} + \frac{A^3b^6}{729a^6} - \frac{2A^9c}{81a^7} - \frac{4A^6b^2c}{81a^6} - \frac{2A^3b^4c}{81a^5} + \frac{4A^6c^2}{27a^5} + \frac{4A^3b^2c^2}{27a^4} - \frac{8A^3c^3}{27a^3} +$$

$$+\left(-\frac{2A^9b}{81a^7} - \frac{4A^6b^3}{81a^6} - \frac{2A^3b^5}{81a^5} + \frac{8A^6bc}{27a^5} + \frac{8A^3b^3c}{27a^4} - \frac{8A^3bc^2}{9a^3}\right)x^3 +$$

$$+\left(\frac{2A^6b^2}{27a^5} + \frac{A^3b^4}{9a^4} + \frac{A^6c}{9a^4} - \frac{4A^3b^2c}{9a^3} - \frac{A^3c^2}{3a^2}\right)x^6 + \left(\frac{A^6b}{9a^4} + \frac{4A^3b^3}{27a^3} - \frac{2A^3bc}{3a^2}\right)x^9$$

Thus, further considerations are devoted to finding the cases, which are interesting for us:

Case 1: $b = 0$. The result has the form

$$\frac{A^{12}}{729a^9} - \frac{2A^9c}{81a^7} + \frac{4A^6c^2}{27a^5} - \frac{8A^3c^3}{27a^3} + \left(\frac{A^6c}{9a^4} - \frac{A^3c^2}{3a^2}\right)x^6$$

Subcase 1.1: $c = 0$. The result gets the form $\frac{A^{12}}{729a^9}$, which is a cube of an integer, thus, it is primitive and not interesting.

Subcase 1.2: $c = \frac{A^3}{3a^2}$, the result is $-\frac{A^{12}}{729a^9}$, again primitive.

Case 2: $c = \frac{3A^3+4ab^2}{18a^2}$. The result:

$$-\frac{A^3b^6}{19683a^6} - \frac{2A^3b^5x^3}{729a^5} + \left(\frac{A^9}{108a^6} - \frac{A^3b^4}{243a^4}\right)x^6$$

Factorizing the coefficient of the last term, one can obtain:

$$-\frac{A^3\left(-3A^3+2ab^2\right)\left(3A^3+2ab^2\right)}{972a^6}$$

Substituting $a = \frac{3A^3}{2b^2}$. Then the result will get the form:

$$-\frac{64b^{18}}{14348907A^6} - \frac{64b^{15}x^3}{177147A^3}$$

Finally, taking $x = \frac{2yb}{A}$ we obtain the result, which is interesting for us: $-1 - 648y^3$.

Practical considerations: Now we investigate this result for applications to solve the process of the equation (4). Since for $max[abs[a, b, c]] \leq 10^{14}$ there are well-known tables in [4], thus, we seek solutions of (4) satisfying the condition $max[abs[a, b, c]] \geq 10^{15}$ with possible small values of $abs[d]$ (desirably less than 1000).

Thus, the result is:

$$\left(54y^2\left(1+36y^3+432y^6\right)\right)^3 - \left(18y^2\left(1+108y^3+1296y^6\right)\right)^3 - \left(1+216y^3+3888y^6\right)^3$$
$$= -1 - 648y^3$$

Sure, calculations were expected to be significantly hard, for which we will use Mathematica 11.0 code:

$$G_8[\text{y\_}] := 1\Big/\text{GCD}\left[\left(54y^2\left(1+36y^3+432y^6\right)\right), \left(18y^2\left(1+108y^3+1296y^6\right)\right),\right.$$
$$\left.\left(1+216y^3+3888y^6\right)\right]$$

$$F_8[\text{y\_}]{:=}G_8[y]\left\{\left(54y^2\left(1+36y^3+432y^6\right)\right), \left(18y^2\left(1+108y^3+1296y^6\right)\right), \left(1+216y^3+3888y^6\right)\right\}$$

$$V_8[\text{y\_}] := G_8[y]^3\left(-1-648y^3\right)$$

$$For[i = -50, i \leq 50, i++$$
$$If[Abs[V_8[i]] < 1000000, If[Max[Abs[F_8[i]]] > 1000000000000, Print[\{i, F_8[i], V_8[i]\}]]]]$$

The result is:

$$\{-11, \{5000250899358, 5000250895002, 6887541673\}, 862487\}$$

$$\{-10, \{2332605605400, 2332605601800, 3887784001\}, 647999\}$$
$$\{-9, \{1004079120606, 1004079117690, 2066085145\}, 472391\}$$
$$\{9, \{1004308703118, 1004308700202, 2066400073\}, -472393\}$$
$$\{10, \{2332994405400, 2332994401800, 3888216001\}, -648001\}$$
$$\{11, \{5000877065646, 5000877061290, 6888116665\}, -862489\}$$

This means that, for instance:

$$1004079120606^3 - 1004079117690^3 - 2066085145^3 = 472391.$$

Result 2: More enhanced result is obtained for the case (9, 9, 7):

$$(3+360y^3+10368y^6+93312y^9)^3 - (-1+216y^3+10368y^6+93312y^9)^3$$
$$-(4y(5+324y^3+3888y^6))^3 = 28+1072y^3$$

$$G_9[y\_] := 1/GCD[(3+360y^3+10368y^6+93312y^9), (-1+216y^3+10368y^6+93312y^9),$$
$$(4y(5+324y^3+3888y^6))]$$
$$F_9[y\_] := G_9[y] * \{(3(1+120y^3+3456y^6+31104y^9)), ((-1+216y^3+10368y^6+93312y^9)),$$
$$(4y(5+324y^3+3888y^6))\}$$
$$V_9[y\_] := G_9[y]^3 * (28+1072y^3)$$
$$For[i=-50, i \leq 50, i++,$$
$$If[Abs[V_9[i]] < 10000000, If[Max[Abs[F_9[i]]] > 1000000000000000, Print[\{i, F_9[i], V_9[i]\}]]]$$

The result is:

$$\{-21, \{-74114970486757701, -74114970485424121, -28010276942676\}, -9927764\}$$
$$\{-20, \{-47775080450879997, -47775080449728001, -19906352640400\}, -8575972\}$$
$$\{-19, \{-30110146685929077, -30110146684941385, -13901324389292\}, -7352820\}$$
$$\{-18, \{-18508949466179901, -18508949465340097, -9521109889128\}, -6251876\}$$
$$\{-17, \{-11065421675141349, -11065421674433881, -6381478799620\}, -5266708\}$$
$$\{-16, \{-6412177868488701, -6412177867898881, -4174623277376\}, -4390884\}$$
$$\{-15, \{-3587108653214997, -3587108652729001, -2657139390300\}, -3617972\}$$
$$\{-14, \{-1927845532267197, -1927845531872065, -1639341027352\}, -2941540\}$$
$$\{14, \{1928001664725699, 1928001664330559, 1639440601624\}, 2941596\}$$
$$\{15, \{3587344849215003, 3587344848728999, 2657270610300\}, 3618028\}$$
$$\{16, \{6412525760839683, 6412525760249855, 4174793146688\}, 4390940\}$$
$$\{17, \{11065922191772139, 11065922191064663, 6381695286052\}, 5266764\}$$
$$\{18, \{18509654743656771, 18509654742816959, 9521381986920\}, 6251932\}$$
$$\{19, \{30111122229317499, 30111122228329799, 13901662181324\}, 7352876\}$$

$$\{20, \{47776407554880003, 47776407553727999, 19906767360400\}, 8576028\}$$

$$\{21, \{74116748933042763, 74116748931709175, 28010781037428\}, 9927820\}$$

Here the most interesting triplet is:

$$1928001664725699^3 - 1928001664330559^3 - 1639440601624^3 = 2941596$$

Result 3: Now, considering the case when $(25, 25, 18)$, using the same approach we obtain:

$$\left(\frac{1}{18}y(63 + 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24})\right)^3$$

$$-\left(\frac{1}{18}y(63 - 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24})\right)^3 - \left(\frac{2}{3}(3 + 20y^6 + 32y^{12} + 32y^{18})\right)^3$$

$$= -8 - 13y^6$$

Thus, one uses the following code:

$$G_{25}[y\_] := 1/GCD[\left(\frac{1}{18}y\left(63 + 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24}\right)\right),$$

$$\left(\frac{1}{18}y\left(63 - 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24}\right)\right), \left(\frac{2}{3}\left(3 + 20y^6 + 32y^{12} + 32y^{18}\right)\right)]$$

$$F_{25}[y\_] := G_{25}[y] * \{\left(\frac{1}{18}y\left(63 + 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24}\right)\right),$$

$$\left(\frac{1}{18}y\left(63 - 36y^3 + 280y^6 + 672y^{12} + 768y^{18} + 512y^{24}\right)\right), \left(\frac{2}{3}\left(3 + 20y^6 + 32y^{12} + 32y^{18}\right)\right)\}$$

$$V_{25}[y\_] := G_{25}[y]^3 * (-8 - 13y^6)$$

$$For[i = -50, i \leq 13, i++,$$

$$If[Abs[V_{25}[i]] < 1000000000, \qquad If[Max[Abs[F_{25}[i]]] > 10000000000000000000,$$

$$Print[\{i, F_{25}[i], V_{25}[i]\}]]]]$$

The result is:

$$\{-28, \{-2147426188301057595107218889007307960 1,$$

$$-21474261883010575951072188890074308913, 1193639792964388519010222081\}, -783071745\}$$

$$\{-24, \{-45524848207155363558693876194364215 4, -45524848207155363558693876194430570 6,$$

$$744442359051171086236385 29\}, -310542337\}$$

$$\{-20, \{-47721859962925526402844543998400 35, -47721859962925526402844544001600 35,$$

$$279620271035733376000000 1\}, -104000001\}$$

$$\{-18, \{-68518855888486826686758454681244 7, -68518855888486826686758454723235 1,$$

$$839390063618729392197122\}, -442158920\}$$

$$\{-16, \{-18028810148480439485764921655324, -18028810148480439485764921786396,$$

$$503719121530094123745 29\}, -27262977\}$$

$$\{-14, \{-12799660023553522713197330140 33, -12799660023553522713197331676 97,$$

$$9106750099297148243458\}, -97883976\}$$

$$\{-13, \{-40143145004075518593772780823 9, -401431450040755185937728036727,$$

$$4798098357335276047604\}, -501988200\}$$

$$\{-12, \{-135674687382873545121755420 37, -135674687382873545121755835 09,$$

$$2839823167678672896 01\}, -4852225\}$$

$$\{-11, \{-616374904448368103731169368 1, -616374904448368103731181080 9,$$

$$2372232726153265249 16\}, -184242408\}$$

$$\{-10, \{-284444871111484444599980035, -284444871111484444600020035,$$

$$2133335466668000000 2\}, -13000008\}$$

$$\{-9, \{-40840534128228425942658291, -40840534128228425942710779,$$

$$64040498230130241 16\}, -55269928\}$$

$$\{-8, \{-537303438740776265183246, -537303438740776265191438,$$

$$192154317110640641\}, -425985\}$$

$$\{-7, \{-762928764093957823651 73, -762928764093957823843 81,$$

$$69479570742237044\}, -12235560\}$$

$$\{-6, \{-808709748243399993333, -808709748243399998517,$$

$$2166658847571458\}, -606536\}$$

$$\{6, \{808709748243399998517, 808709748243399993333,$$

$$2166658847571458\}, -606536\}$$

$$\{7, \{762928764093957823843 81, 762928764093957823651 73,$$

$$69479570742237044\}, -12235560\}$$

$$\{8, \{537303438740776265191438, 537303438740776265183246,$$

$$192154317110640641\}, -425985\}$$

$$\{9, \{40840534128228425942710779, 40840534128228425942658291,$$

$$64040498230130241 16\}, -55269928\}$$

$$\{10, \{284444871111484444600020035, 284444871111484444599980035,$$

$$2133335466668000000 2\}, -13000008\}$$

$$\{11, \{616374904448368103731181080 9, 616374904448368103731169368 1,$$

$$2372232726153265249 16\}, -184242408\}$$

$$\{12, \{135674687382873545121755835 09, 135674687382873545121755420 37,$$

$$2839823167678672896 01\}, -4852225\}$$

$$\{13, \{401431450040755185937728036727, 401431450040755185937727808239,$$

$$4798098357335276047604\}, -501988200\}$$

Here the most interesting triple is:

$$(-2147426188301057595107218889007307960 1)^3 + (2147426188301057595107218889007430891 3)^3$$

$$-(11936397929643885190102220 81)^3 = -783071745$$

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$a^3 + b^3 + c^3 = d$ տեսքի դիոֆանտյան հավասարման լուծման նոր մեթոդ

Ա. Ավագյան

Ամփոփում

Հոդվածը նվիրված է հայտնի $a^3 + b^3 + c^3 = d$տեսքի դիոֆանտյան հավասարումների որոշ մասնավոր դեպքերի լուծմանը:Կիրառելով “Mathematica 11” փաթեթը, հաջողվել է գտնել նշված խնդրի լուծման արդյունավետ ալգորիթմ, որն այս խնդրի լուծման այլ ալգորթմների համեմատ ավելի պարզ է և օգտագործում է զգալիորեն փոքր քանակությամբ գործողություններ:

Новый метод решения Диофантого уравнения $a^3 + b^3 + c^3 = d$

А. Авагян

Аннотация

Статья посвящена решению известного диофантового уравнения вида $a^3+b^3+c^3 = d$ в некоторых частных случаях. Используя пакет "Mathematica 11" удалось найти эффективный алгоритм нахождения решений этой задачи, который по сравнению с другими известными алгоритмами, дающими решения этой задачи, является более простым и использует значительно меньшее количество операций.